CH. 9.1 – 9.3 STRESS TRANSFORMATIONS

STRESS AT A POINT

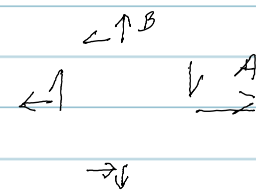
FIND MAX. STRESS

3D $\sigma = Ai + Bj + Ck$ (KSI)

$\tau = Di + Ej + FK$ (KSI)

2D $\sigma = Ai + Bj$

$\tau = Ci + Dj$

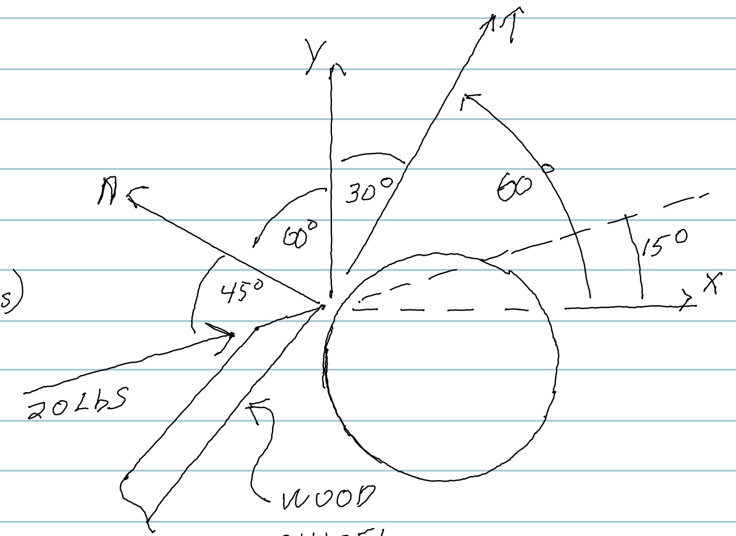


TRANSFORMATION OF AXIS

DO THIS FOR FORCES

BREAK INTO X+Y COMP.

$F =$ ______ $i +$ ______ j (lbs)



BREAK INTO N+T COMP.

$F =$ ______ $i_n -$ ______ j_n (lbs)

☐ USE TRANSFORMATION EQUATION TO FIND EQUIVALENT VECTOR COMPONENTS USING A ROTATED AXIS BY θ. NEW AXIS x', y' or n, τ IN THIS CAS

$x' = x\cos(\theta) + y\sin(\theta)$ $\quad y' = -x\sin(\theta) + y\cos(\theta)$

SUB. IN: $x = \underline{19.3}$, $y = \underline{5.18}$, $\theta = 60°$

$x' = \underline{14.1}$ $\quad y' = \underline{-14.1}$ or $\underline{\tau = 14.1,\ n = -14.1}$

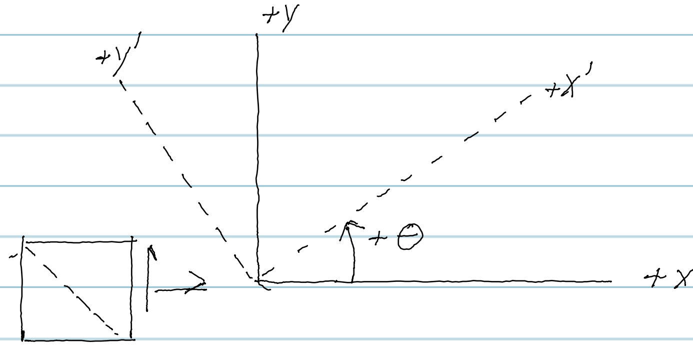
SUB IN: $x = 19.3$ $\quad y = 5.18$ $\quad \theta = 15°$

$\underline{x' = 20}$ $\quad \underline{y' \approx 0}$ PRINCIPAL AXIS!!

CH 9.1 – 9.3 STRESS TRANSFORMATIONS (CONT.)

GIVEN:

$\sigma_x,\ \sigma_y,\ \tau_{xy},\ \theta$



$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

MATHCAD

STRESS TRANSFORMATIONS.mcd

$$\tau_{x'y'}' = -\frac{\sigma_x - \sigma_y}{2}\sin(2\theta) + \tau_{xy}\cos(2\theta)$$

$$\sigma_{y'}' = \frac{\sigma_x - \sigma_y}{2}\sin(2\theta) + \tau_{xy}\cos(2\theta)$$

SIGN CONVENTIONS

DRAW +X + +X' AS POSITIVE (OUTWARD + NORMAL) FROM ELEMENT.

1) σ_x + σ_x' ARE POSITIVE POINTING AWAY FROM ELEMENT.

2) τ_{xy} & $\tau_{x'y'}'$ ARE POSITIVE POINTING IN ~~X~~ ~~OR~~ +Y DIRECTION.

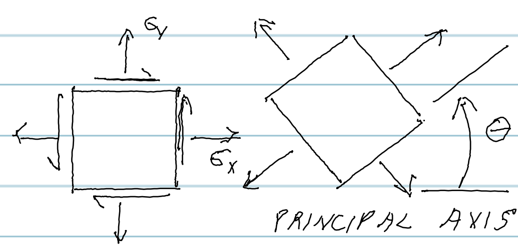
3) ANGLE θ MEASURED FROM +X AXIS TO +X' AXIS (CCW DIRECTION)

SECTION 9.3 PRINCIPAL STRESSES

$$\sigma_{1,2} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} + \frac{\sigma_x + \sigma_y}{2} \qquad \sigma_1 > \sigma_2,$$

$$\tan(2\theta_{ps}) = \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)}$$

2 ROOTS
90° APART



PRINCIPAL AXIS
// STRESSES

TAKE AWAY – 1) FIND MAX. σ + τ

CH. 9.1 - 9.3 STRESS TRANSFORMATIONS (CONT.)

MAXIMUM SHEAR STRESSES

EXIST AT 45° FROM PRINCIPAL STRESSES

IMPORTANT IN DUCTILE MATERIAL FAILURES

NORMAL STRESSES EXIST ON THIS PLANE

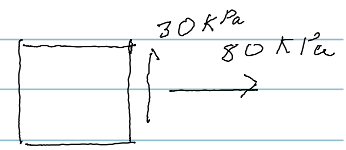
$$\tau_{\substack{max \\ \text{IN-PLANE}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$ MAXIMUM IN-PLANE SHEAR STRESS

$$\tan(2\theta_{\tau}) = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$ τ_{max} PLANE ANGLE

$$\sigma_{AVG} = \frac{\sigma_x + \sigma_y}{2}$$ NORMAL STRESSES EXIST AN. @ τ_{max} PLANE

EXAMPLE

$\sigma_x = 80\ kPa$, $\tau_{xy} = 30\ kPa$



FIND: PRINCIPAL STRESSES $\sigma_x' = ?$, $\sigma_y' = ?$, $\theta_p = ?$

SOLUTION

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan(2\theta_p) = \frac{\tau_{xy}}{\frac{(\sigma_x - \sigma_y)}{2}}$$

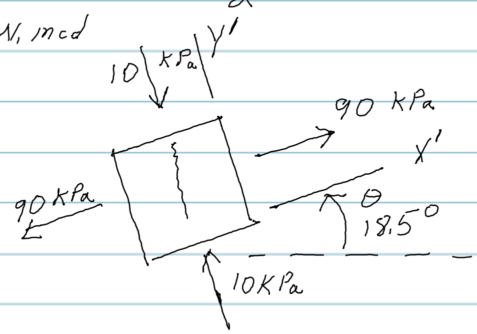
$\sigma_1 = \underline{\underline{90\ kPa}}$

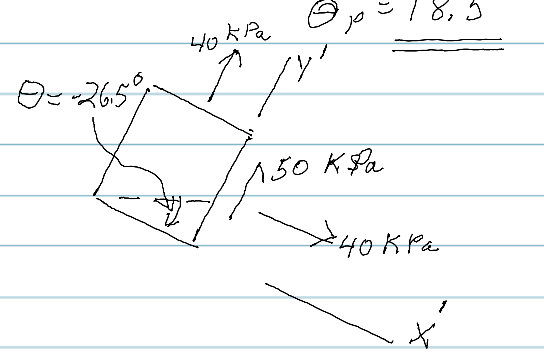
$\sigma_2 = \underline{\underline{-10\ kPa}}$

$\theta_p = \underline{\underline{18.5^\circ}}$

MATHCAD

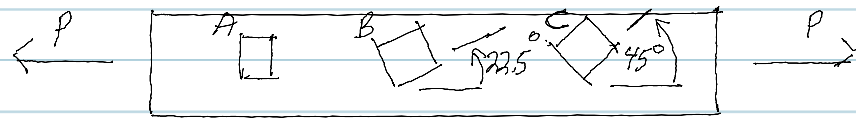
STRESS TRANSFORMATION.mcd





RANKING STRESS TRANSFORMATIONS

THREE STRESS ELEMENTS IN DIFFERENT ORIENTATIONS WITH RESPECT TO THE AXIAL LOADED MEMBER ARE SHOWN. RANK THESE THREE ELEMENTS ON THE BASIS OF THE SHEAR STRESS IN EACH ELEMENT. ASSUME ALL 3 POINTS (ELEMENTS) ARE AT THE SAME CTR. LINE LOCATION ON THE MEMBER.



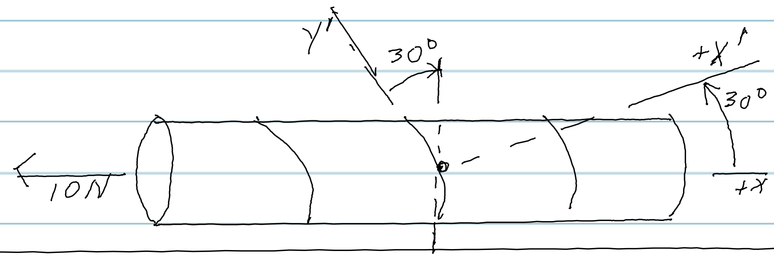
GREATEST 1 ___ 2 ___ 3 ___ LEAST

EXPLAIN YOUR REASONING.

CH 9.1 – 9.3 PROB. 9 – 38/39 STRESS

GIVEN: PAPER TUBE ROLLED FROM STRIP OF PAPER 1 mm THICK WITH A 30 mm O.D. THE GLUE SEAM RUNS AT AN ANGLE OF 30° FROM THE VERTICAL. AXIAL F = 10 N

SKETCH



FIND: a) SHEAR STRESS ALONG GLUE SEAM $\tau'_{xy} = ?$
b) NORMAL STRESS ⊥ TO GLUE SEAM $\sigma'_x = ?$

SOLUTION

1) FIND REACTIONS @ C.S.
2) NAME COMPONENT REACTIONS (V, M_b, T, N) ONLY $N + V + M_b$ – 2D
3) SOLVE FOR σ_x, σ_y, τ_{xy}, M_b (2 D ONLY)
4) TRANSLATE TO GLUE LINE AXIS TO GET σ'_y + τ'_{xy}

1) $P = F = 10\ N$

2) $N = 10\ N$

3) $\sigma_x = \sigma_N = \dfrac{P}{A} = \dfrac{10\ N}{9.111 \cdot 10^{-5}\ m^2} = \underline{\underline{109.8\ kPa}}$ $A_{CS} = \dfrac{\pi}{4}\left(d_{out}^2 - d_{in}^2\right)$

INPUT: $\sigma_x = 109.8\ kPa$, $\sigma_y = 0$, $\tau = 0$, $\theta = 30°$ $A_{CS} = \dfrac{\pi}{4}\left(.03\ m^2 - .028\ m^2\right)$

4) $\tau'_{xy} = \dfrac{-(\sigma_x - \sigma_y)}{2} \sin(2\theta) + \tau_{xy} \cos(2\theta)$ $A_{CS} = 9.111 \cdot 10^{-5}\ m^2$

$\sigma'_y = \dfrac{\sigma_x + \sigma_y}{2} - \dfrac{\sigma_x - \sigma_y}{2} \cos(2\theta) - \tau_{xy} \sin(2\theta)$

$\underline{\underline{\tau'_{xy} = -47.5\ kPa = 6.89\ psi = 7\ psi}}$

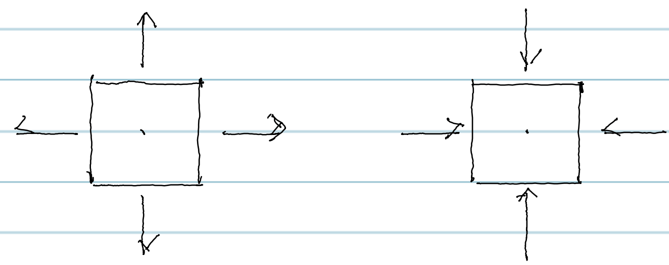
$\underline{\underline{\sigma'_x = 82.3\ kPa = 11.9\ psi = 12\ psi}}$

CH. 9.5 ABSOLUTE MAXIMUM SHEAR STRESS (OUT OF PLANE)

3-D STRESS (SHEAR)

IF BIAXIAL STRESSES OF SAME SIGN

□ $\tau_{\substack{ABS\\MAX}} = \left|\frac{\sigma_1}{2}\right|$ or $\left|\frac{\sigma_2}{2}\right|$

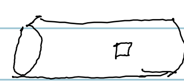


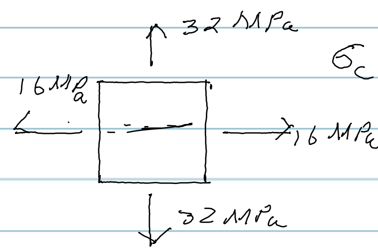
IF BIAXIAL STRESSES OPPOSITE SIGN – NOT ANY LARGER THAN $\tau'_{\substack{MAX\\IN\text{-}LINE}}$

□ $\tau_{\substack{ABS\\MAX}} = \frac{\sigma_1 - \sigma_2}{2} = \tau'_{\substack{MAX\\IN\text{-}PLANE}}$

EXAMPE CYLINDER PRESSURE VESSEL

$\tau_{\substack{max\\IN\text{-}LINE}} = \underline{\underline{8\ MPa}}$





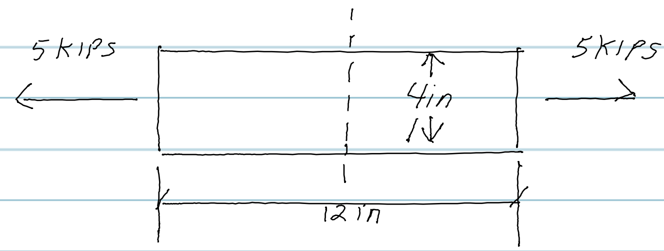
$\sigma_c = 2\sigma_L$

$\tau_{\substack{ABS\\MAX}} = \frac{32\ MPa}{2} = \underline{\underline{16\ MPa}}$

CH 9.5 PROB. 9.86

GIVEN A RECTANGULAR PLATE 12" LONG x 4" WIDE x 1/2" THICK IS LOADED WITH 5 KIPS OF FORCE.

SKETCH



FIND PRINCIPAL STRESSES $\sigma_1 = ?$, $\sigma_2 = ?$, + $\tau_{\substack{ABS\\MAX}} = ?$

SOLUTION

□ $\sigma_x = \dfrac{N}{A} = \dfrac{5 \text{ KIPS}}{(4in \cdot \frac{1}{2}'' in)} = \dfrac{5 \text{ KIPS}}{2 \, in^2} = \underline{\underline{2.5 \text{ KSI}}}$

□ $\underline{\underline{\sigma_x = 2.5 \text{ ksi}}}$, $\underline{\underline{\sigma_y = 0}}$, $\tau_{xy} = 0$ $\theta = 0$

$\tau_{\substack{ABS\\MAX}} = \cancel{\sigma}_{\substack{ABS\\MAX}} = \dfrac{\sigma_1}{2}$ or $\dfrac{\sigma_2}{2} = \dfrac{2.5 \text{ ksi}}{2} = \underline{\underline{1.25 \text{ ksi}}}$

□ $\sigma_{\substack{MAX\\IN\text{-}LINE}} = \underline{\underline{1.25 \text{ Ksi}}}$ $\tau_{\substack{max\\INPLANE}} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$