

CH. 6 BENDING 6.1-6.2

SECTION 6.1

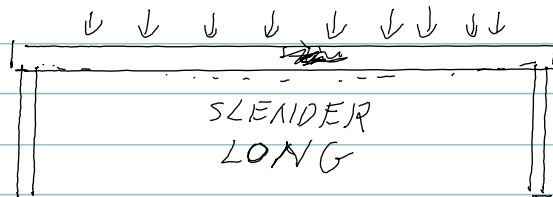
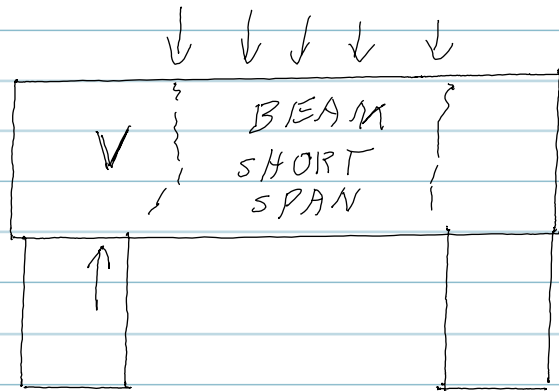
FAILURE OF BEAMS

CH 1.2 SEE SMARTNOTES
Pg 8-9.

NORMAL - DIRECTION
OF LONG AXIS

SHEAR "V" - ACROSS

BENDING MOMENT



DRAW/SKETCH BENDING MOMENT DIAGRAMS

$x < \frac{1}{2}L$

$\sum F_y = 0$
 $+\frac{1}{2}P - V = 0$
 $V = \frac{1}{2}P$
 $\sum M = \sum F \cdot r_{\perp} = 0$
 $-\frac{1}{2}Px + M = 0$
 $M = \frac{1}{2}Px$
 $y = mx + b$

$x > \frac{1}{2}L$

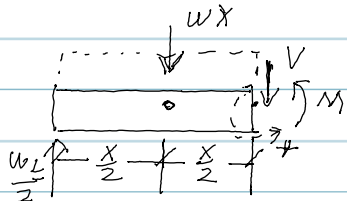
$\sum F_y = 0$
 $+\frac{1}{2}P - P - V = 0$
 $-\frac{1}{2}P - V = 0$
 $V = -\frac{1}{2}P$
 $\sum M = \sum F \cdot r_{\perp} = 0$
 $-\frac{1}{2}Px + P(x - \frac{1}{2}L) + M = 0$
 $-\frac{1}{2}Px + Px - \frac{1}{2}PL + M = 0$
 $\frac{1}{2}Px - \frac{1}{2}PL + M = 0$
 $M = -\frac{1}{2}Px + \frac{1}{2}PL$
 $M_{x=\frac{1}{2}L} = -\frac{1}{2}P(\frac{1}{2}L) + \frac{1}{2}PL$
 $M_{\frac{1}{2}L} = +\frac{1}{4}PL$

$A_1 = \frac{1}{2}P \cdot \frac{1}{2}L$
 $A_1 = +\frac{1}{4}PL$
 $-\frac{1}{4}PL$
 $\frac{1}{4}PL$

CH. 6.1 - 6.2 BENDING (CONT.)

SECTION 6.2

SIMPLE BEAM
UNIFORM LOAD



$$\sum F_y = 0 \Rightarrow +\frac{wL}{2} - wx - V = 0$$

$$V(x) = -wx + \frac{wL}{2}$$

$$y = mx + b$$

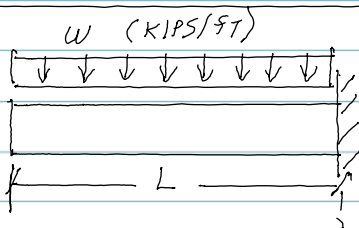
$$\sum M = 0 \Rightarrow -\frac{wx}{2} \cdot x + (wx) \frac{x}{2} + M = 0$$

$$M(x) = -\frac{1}{2}wx^2 + \frac{wLx}{2}$$

$$M(x) = \int V(x) dx = \int \left(\frac{1}{2}w - wx + \frac{wL}{2} \right) dx$$

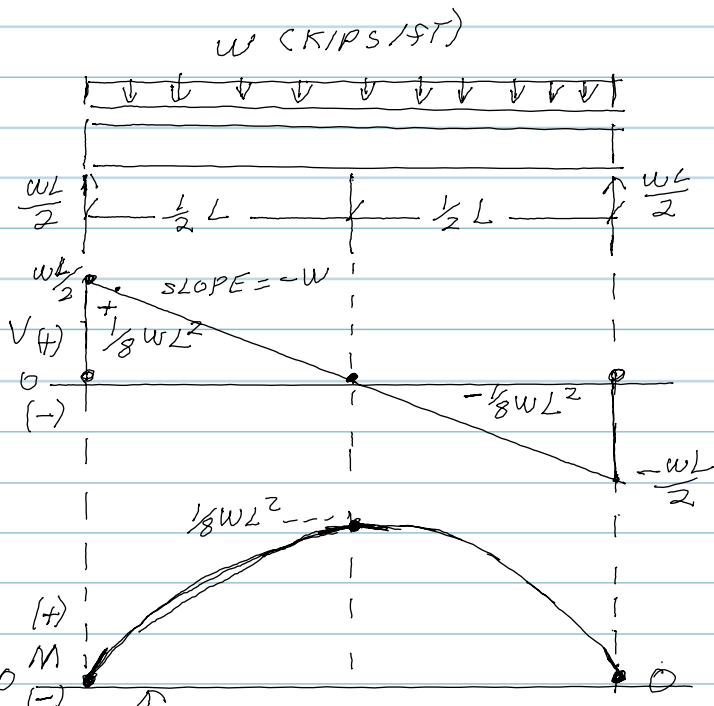
$$M(x) = -\frac{1}{2}wx^2 + \frac{wLx}{2}$$

CANTILEVER BEAM



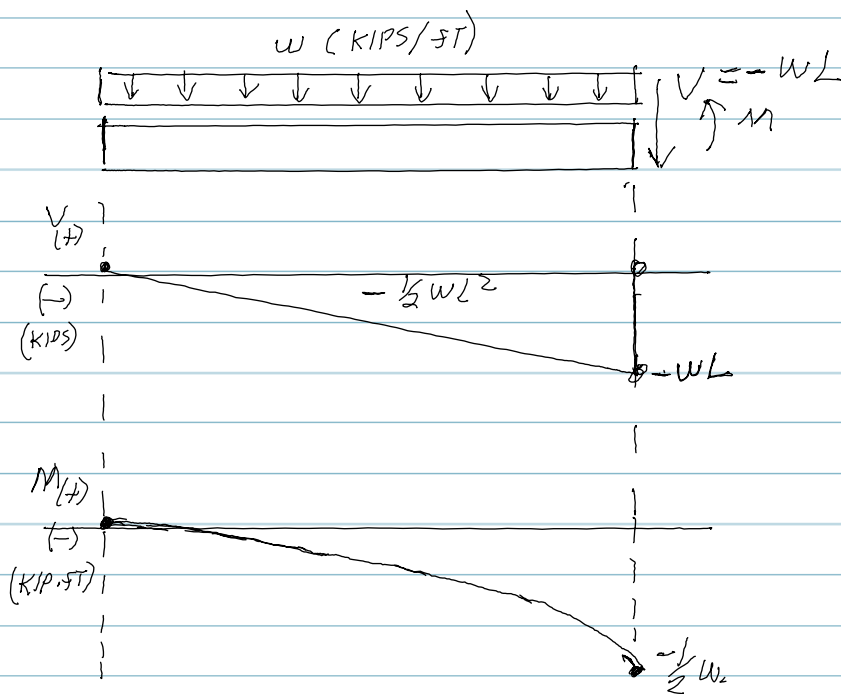
TAKE AWAY

V_{MAX} @ WALL (wL)
 M_{MAX} @ WALL ($\frac{1}{2}wL^2$)



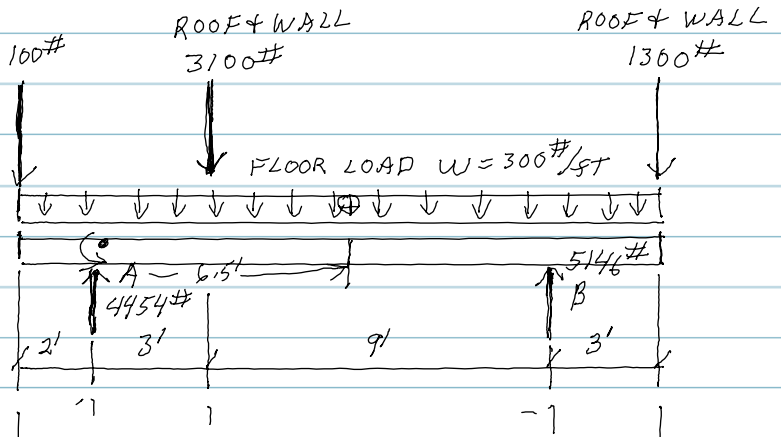
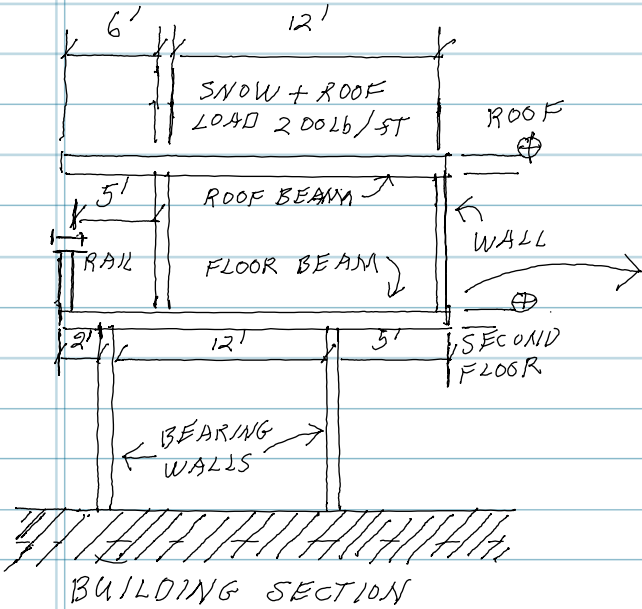
KEY POINTS

- 1) V MAX @ SUPPORTS.
- 2) M MAX @ CTR $\frac{1}{8}wL^2$



CH. 6.1-6.2 BENDING (CONT.)

PRACTICAL EXAMPLE - BEAM DESIGN



$$\sum M_A = 0$$

$$+100(2) - 300(17)6.5 - 3100 + B(-1300) = 0$$

$$-61750 + 12B = 0$$

$$B = 5146 \text{ Lbs}$$

$$\sum F_y = 0$$

$$-100 - 3100 - 1300 + A + 5146 + 300(17) = 0$$

$$A = 4454 \text{ Lbs}$$

$$\text{AREA} = \frac{1}{2}(h_1 + h_2)b$$

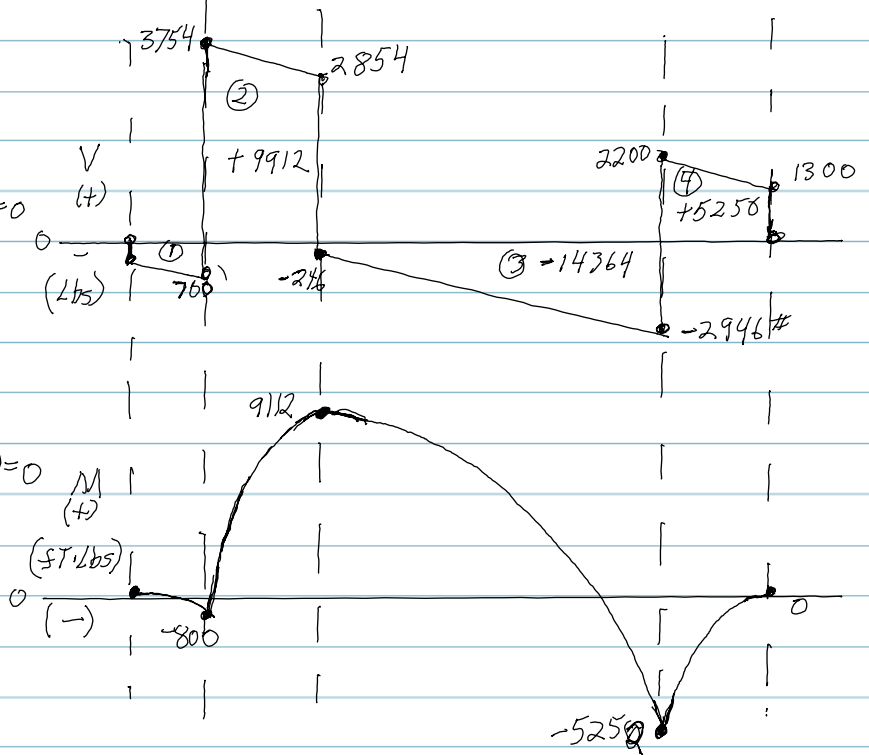
$$A_1 = \frac{1}{2}(2')(100 + 700)$$

$$A_1 = -800 \text{ FT} \cdot \text{Lbs}$$

$$A_2 = \frac{1}{2}(3754 + 2854)3' = 9912$$

$$A_3 = -\frac{1}{2}(2946 + 2946)9' = -14364$$

$$A_4 = +\frac{1}{2}(2200 + 1300)3' = +5250$$



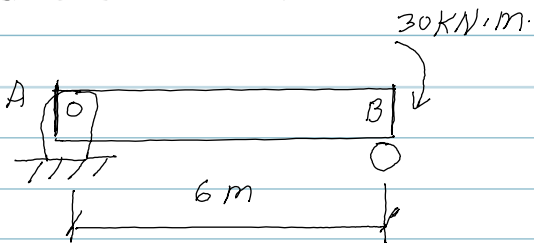
KEY POINTS

- i
- 1) V_{max} @ column "A" 5146 Lbs
- M_{max} @ 3' right of "A" 912 FT · Lbs

CH 6.1-6.2 BENDING (CONT.)

HANDLING APPLIED MOMENTS ON SHEAR & BENDING MOMENT DIAGRAMS

PROB. F6.1



$$\sum M_B = 0$$

$$+(A_y)6 - 30 = 0$$

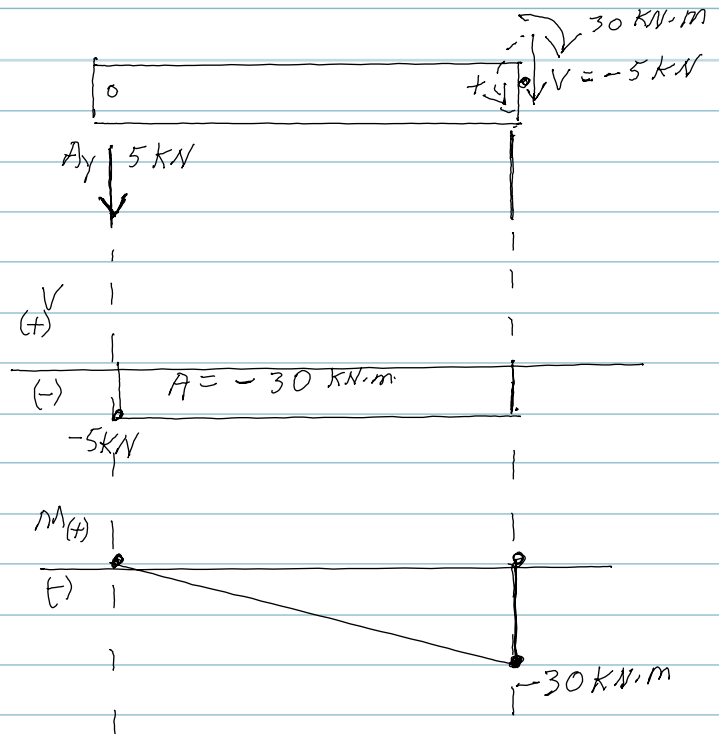
$$A_y = \frac{30}{6} = 5 \text{ kN}$$

$$\sum F_y = 0$$

$$-5 \text{ kN} - V = 0$$

$$V = -5 \text{ kN}$$

FBD



RULE ON MOMENTS - APPLIED

↪ CW CAUSES A JUMP POSITIVE,

FBD & SYM DIAGRAM!

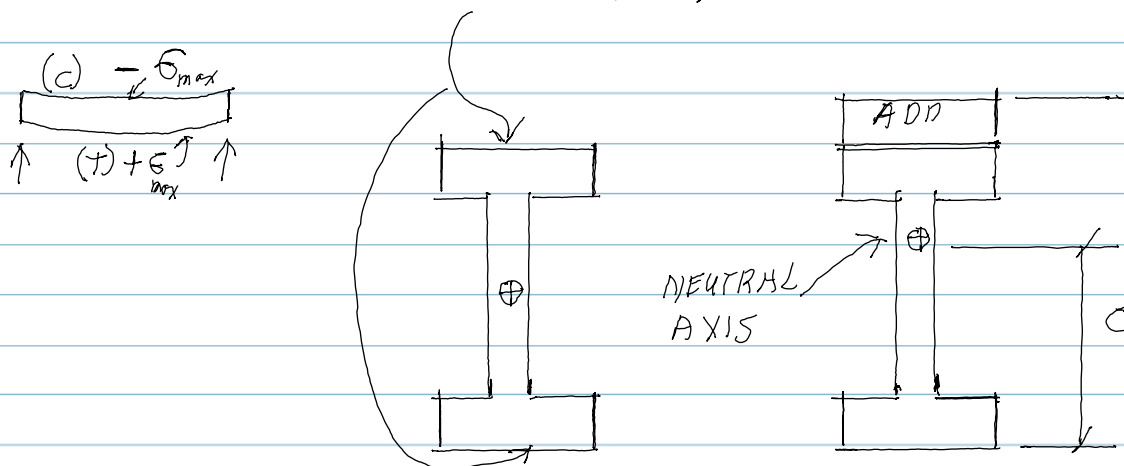
CH. 6.4 THE FLEXURE FORMULA

KEY FORMULA

$$\sigma_{max} = \frac{M c}{I}$$

BENDING MOMENT IN BEAM (FT-LBS, Nm)
 MAX. \perp DISTANCE FROM NEUTRAL AXIS TO BENDING EDGE
 MOMENT OF INERTIA OF CROSS SECTION AT CENTROID (ON NEUTRAL AXIS)
 NORMAL STRESS (MAX).

FLEXURE FORMULA



"M" SHEAR + MOMENT DIAGRAM — SPAN + LOADS — DESIGN PROCESS

σ_{MAX} MATERIAL PROPERTY — ASD

I PROPERTY OF SHAPE — SIZE OF BEAM

PICK ANY TWO $\Rightarrow M, \sigma_{max} (\sigma_{allow})$

CALCULATE I NECESSARY $I \geq 10 \text{ in}^4$ PICK $I = 12.7 \text{ in}^4$

$$\tau_{max} = \frac{T c}{J}$$

CH. 6.4 FLEXURE FORMULA (CONT.)

EXAMPLE PROB. Pg 294, EX 6.12

WIDE FLANGE I BEAM
W 460 x 74 (APPENDIX 'B' Pg 808)
 $I = 333 \cdot 10^6 \text{ mm}^4$

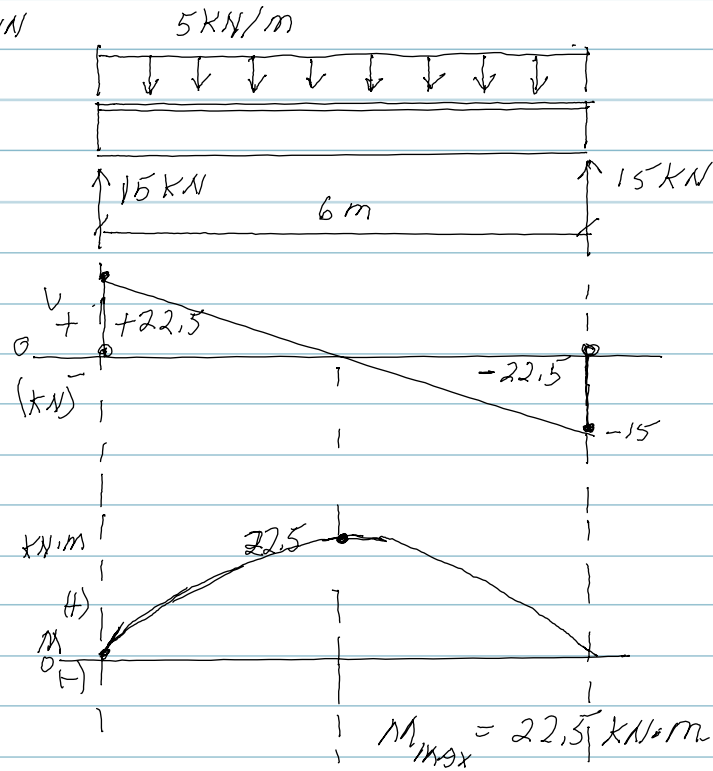
GIVEN: STEEL 'I' BEAM, UNIFORM LOAD
SIMPLE SUPPORTS, SHAPE SHOWN

FIND: $\sigma_{max} = ?$

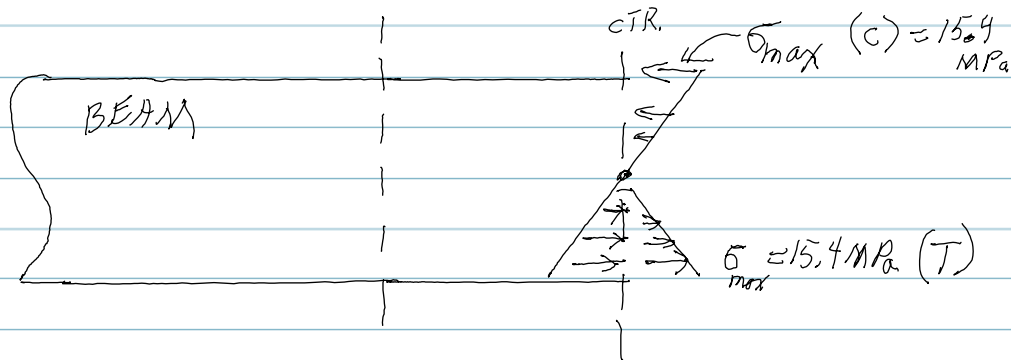
$$\sigma_{max} = \frac{MC}{I} \quad M = 22.5 \text{ kN}\cdot\text{m}$$

$$\sigma_{max} = \frac{(22.5 \cdot 10^3 \text{ N}\cdot\text{m}) \left(\frac{0.437 \text{ m}}{2} \right)}{333 \cdot 10^6 \text{ m}^4}$$

$$\sigma_{max} = 15.4 \text{ MPa}$$



SEE Pg 46 (SN) $M = \frac{1}{8} wL^2$

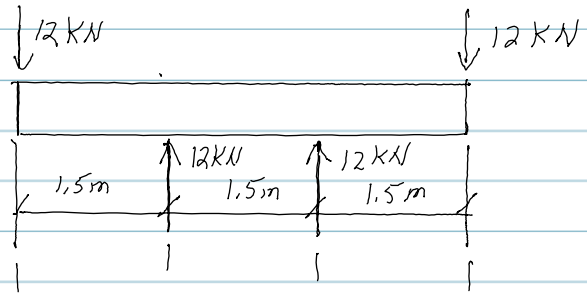
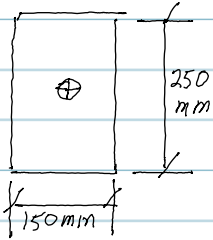


CH 6.4 PROBLEM 6-95

GIVEN:

WOOD BEAM,

FIND: $\sigma_{max} = ?$

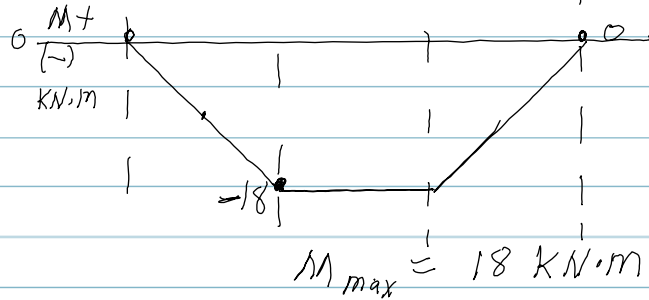
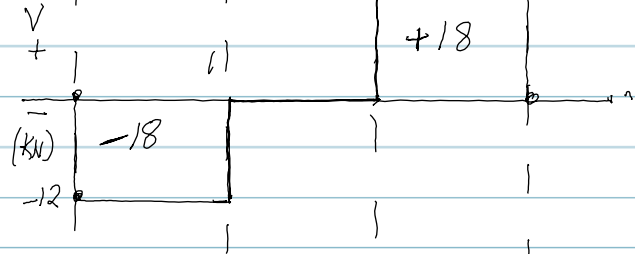


SOLUTION:

$$\sigma_{max} = \frac{Mc}{I}$$

$$\sigma_{max} = \frac{(18 \cdot 10^3 \text{ N}\cdot\text{m})(0.125 \text{ m})}{1.95 \cdot 10^{-4} \text{ m}^4}$$

$$\sigma_{max} = 1.15 \cdot 10^7 \text{ Pa} = 11.5 \text{ MPa}$$



$$c = \frac{0.250 \text{ m}}{2} = 0.125 \text{ m}$$

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (0.15 \text{ m})(0.25 \text{ m})^3$$

$$I = 1.95 \cdot 10^{-4} \text{ m}^4 =$$

CH 6.4 FLEXURE FORMULA PROB. 6-73

GIVEN: SOLID STEEL SHAFT
 SUPPORT @ "A" & "B" (BEARINGS)
 $\sigma_{ALLOW} = 22 \text{ KSI}$

FIND: $d_{min} = ?$

SOLUTION: $\sigma_{max} = \frac{M C}{I}$

1) FBD REACTIONS

$$\sum M_A = 0$$

$$-400(12) + B(30) - 300(45) = 0$$

$$B = 610 \#$$

$$\sum F_y = 0$$

$$A - 400 + 610 - 300 = 0$$

$$A = 90 \#$$

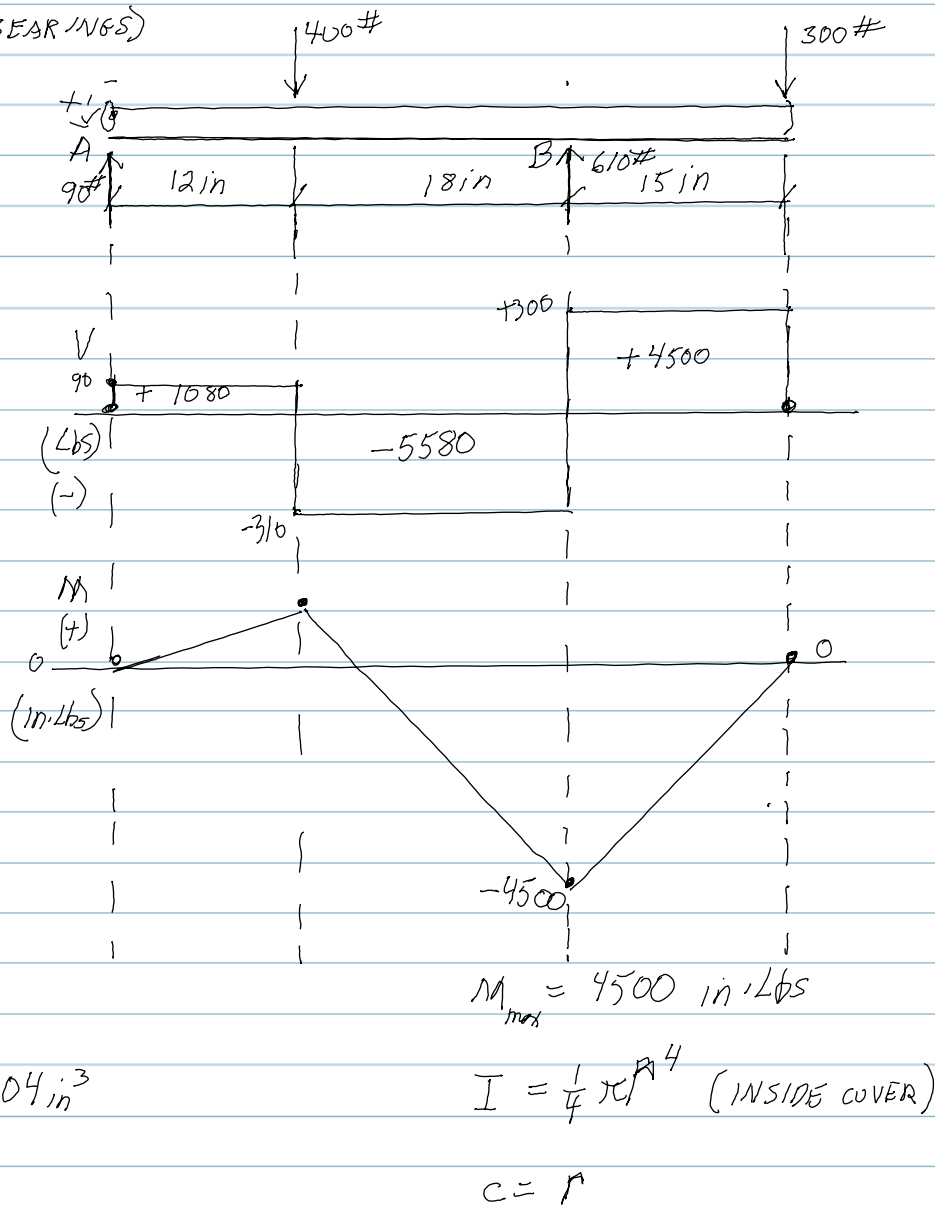
$$2) \sigma_{max} = \frac{M C}{I} = \frac{M}{\frac{1}{4} \pi r^4} = \frac{4 M}{\pi r^4}$$

$$r^3 = \frac{4 M}{\pi \sigma_{max}}$$

$$r^3 = \frac{4(4500 \text{ in} \cdot \text{lbs})}{\pi(22000 \frac{\text{lb}}{\text{in}^2})} = .2604 \text{ in}^3$$

$$r = 0.6386 \text{ in}$$

$$d_{min} = 1.28 \text{ in}$$



CH. 6 SHEAR & BENDING MOMENT CONSTRUCTION PUZZLE

CONSTRUCT THE LOADS & SUPPORT FORCES REQUIRED TO MATCH THE SHEAR DIAGRAM GIVEN. NOTE THE FORCE UNITS ARE KIPS. DISTANCE IS IN UNITS OF FEET.

