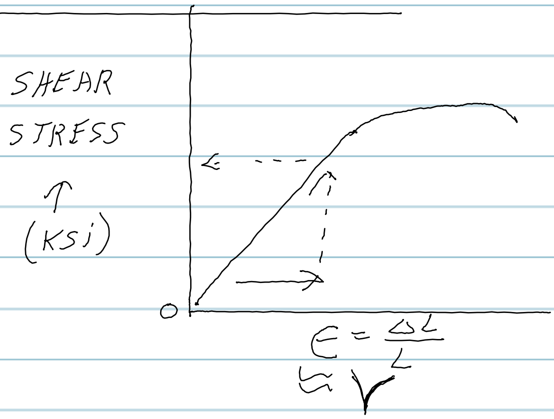
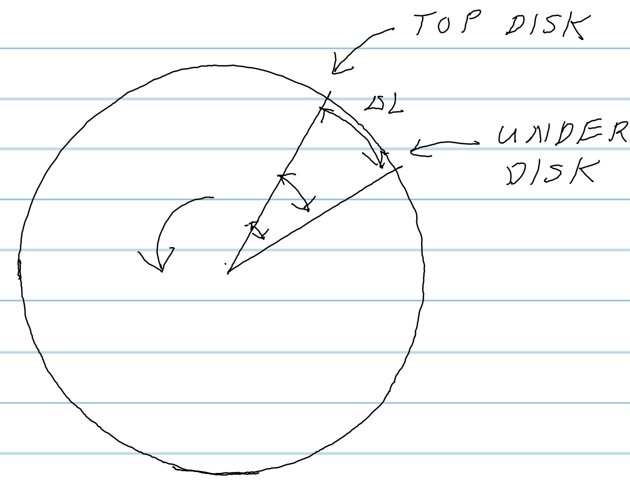
CH. 5.1 – 5.2 – 5.3 TORSION & MAXIMUM SHEAR STRESS

SECTION: 5.1 – 5.2





τ_{max} OCCURS AT OUTER EDGE OF SHAFT.

$$\tau_{max} = \frac{T \cdot C}{J}$$

TORQUE (N·m, FT·lbs)

RADIUS OF THE SHAFT (m, FT)

POLAR MOMENT OF INERTIA (in^4, m^4, mm^4)

SHEAR STRESS (KSI, PSI, P_a, ETC)

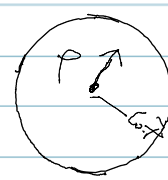
POLAR MOMENTS

RADIUS

SOLID SHAFT: $J = \frac{\pi}{2}(c^4)$

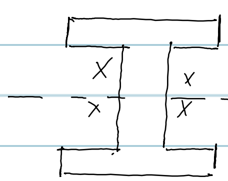
HOLLOW ": $J = \frac{\pi}{2}(c^4 - r_{INSIDE}^4)$

SHEAR STRESS @

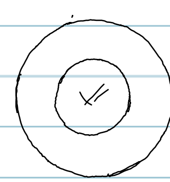


$\rho \le c$

$$\tau = \frac{T\rho}{J} \quad \rho \le c$$



$I_x = \int r^2 da$



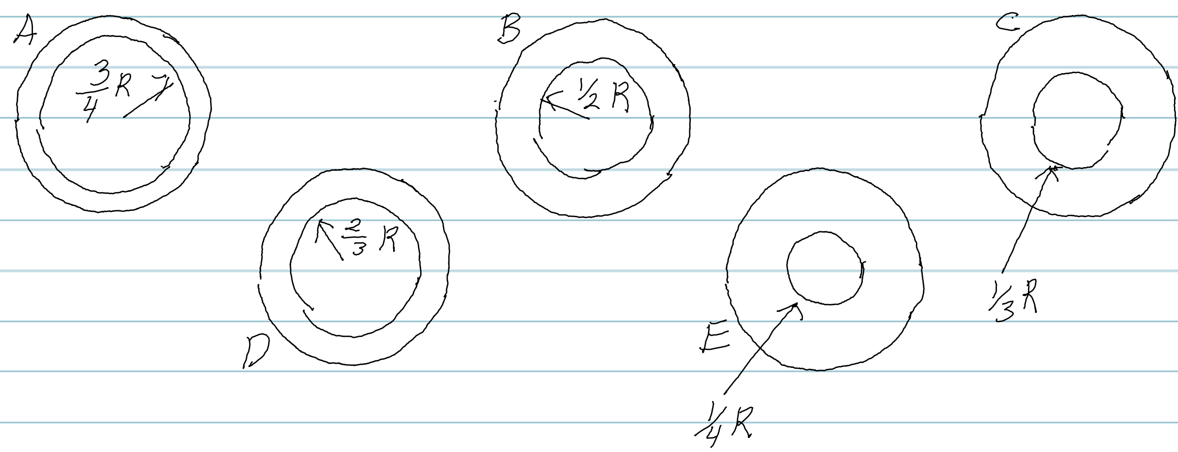
$J_z = \int r^2 da$

CH. 5.3 POWER TRANSMISSION

HORSEPOWER TORQUE
EQUATION

POLAR MOMENT OF INERTIA RANKING

RANK THE 5 OBJECTS (A–E) BASED ON THE SIZE OF THEIR POLAR MOMENT "J". ALL OBJECTS HAVE THE SAME OUTSIDE DIAMETER R. THE THICKNESS OF THE WALL SECTIONS VARY.



LARGEST 1 _____ 2 _____ 2 _____ 4 _____ 5 _____ SMALLEST

ALL POLAR MOMENTS ARE EQUAL. _____

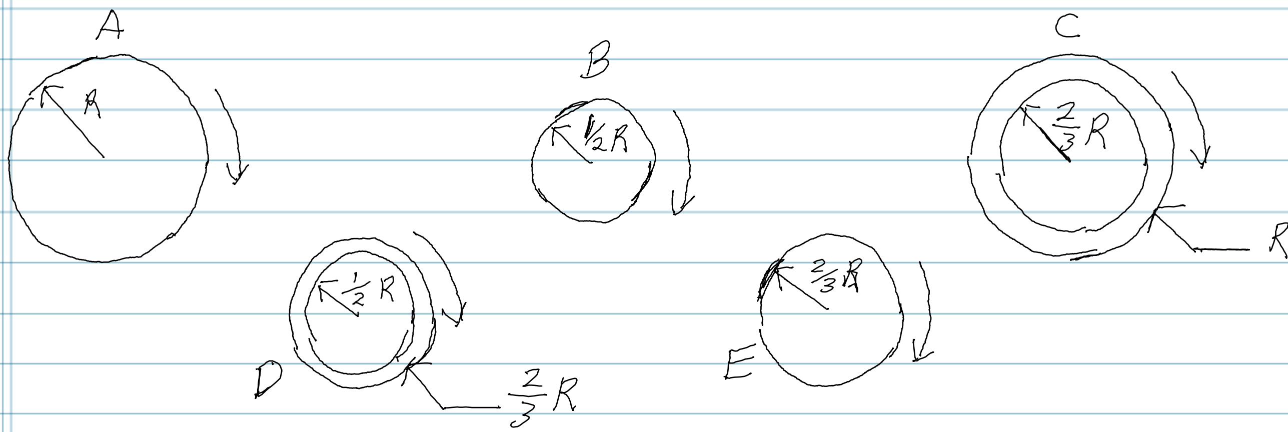
EXPLAIN YOUR REASONING:

CONFIDENCE?

LOW GOOD GREAT (CIRCLE ONE)

SHEAR STRESS IN SHAFTS RANKING

ALL 5 SHAFTS HAVE THE SAME TORQUE "T" APPLIED. RANK THE MAXIMUM SHEAR STRESS IN EACH SHAFT.



LARGEST 1 ___ 2 ____ 3 ___ 4 ____ 5 ____ SMALLEST

EXPLAIN YOUR REASONING;

CONFIDENCE?

LOW FAIR GREAT (CIRCLE ONE)

CH 5.3 PROBLEM 5-37

GIVEN: BOAT PROPELLER DRIVE SHAFT - RPM = 1500, HP = 1800
SHAFT LENGTH = 8 FT, dia = 4 in, SOLID SHAFT

FIND: $\tau_{max} = ?$

SOLUTION: $\tau_{max} = \frac{T c}{J}$ $J = \frac{\pi}{2} c^4 = \frac{\pi}{2}(2 in)^4 = 25.13 in^4$
$c = 2 in$

$T = ?$

$P_{HP} = \frac{T \cdot RPM}{63025} \Rightarrow$

$T = \frac{63025 \cdot P}{RPM} = \frac{63025 \cdot 1800}{1500} = 75,630. \ in \cdot lbs$

$\tau_{max} = \frac{75630 \ in \cdot lbs \cdot 2 in}{25.13 in^4} = \underline{\underline{6018 \ psi = 6.02 \ ksi}}$

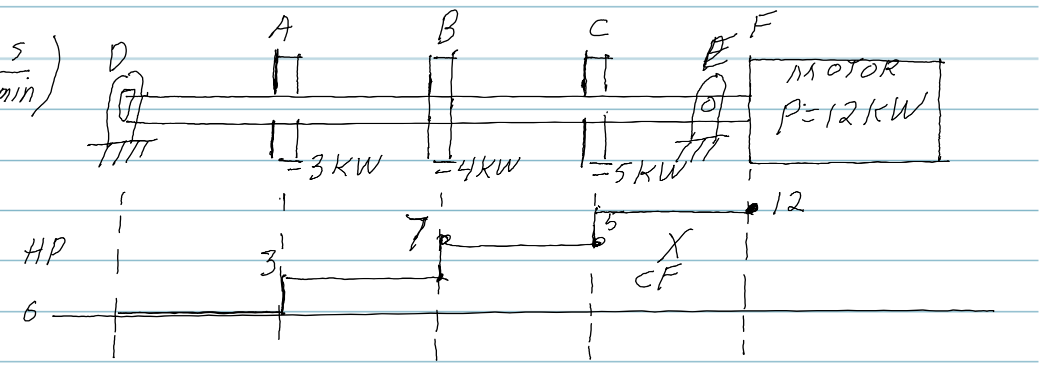
CH 5-3 PROBLEM 5-40

GIVEN: 12 KW ELECTRIC MOTOR TURNS 25 mm dia. SOLID SHAFT THAT HAS 3 OUTPUT GEARS ALONG THE SHAFT. EACH GEAR REMOVES THE GIVEN AMOUNT OF POWER FROM THE SHAFT.

SKETCH

$$w = 50 \frac{rev}{s}\left(\frac{60 s}{1 min}\right)$$

$$w = 3000 \; RPM$$



SOLUTION: $\tau_{max} = ?$

$$\tau_{max} = \frac{T \cdot c}{J}, \quad J = \frac{\pi}{2}(c)^4, \Rightarrow \tau_{max} \simeq T, \quad P = \frac{T \cdot RPM}{9550}, \quad P \simeq T$$

$\tau_{max} \simeq T \simeq P \Rightarrow P \simeq \tau_{max}$ NOTE! ONLY WHEN J IS CONSTANT

$$P = \frac{T \cdot RPM}{9550} \Rightarrow T = \frac{9550 \cdot P}{RPM} = \frac{9550 \cdot 12 KW}{3000 RPM} = \underline{38.2 \; N \cdot m}$$

$$J = \frac{\pi}{2}(c^4) = \frac{\pi}{2}(.0125 m)^4 \qquad c = \frac{d}{2} = \frac{25mm}{2}\left(\frac{1 \; m}{1000 \; mm}\right) = .0125 \; m$$

$$J = 3.835 \cdot 10^{-8} \; m^4$$

$$\tau_{max_{CF}} = \frac{T \cdot C}{J} = \frac{(38.2 \; N \cdot m) \; .0125 \; m}{3.835 \cdot 10^{-8} \; m^4} = 1.25 \cdot 10^7 \; Pa\left(\frac{1 \; MPa}{10^6 \; Pa}\right)$$

$$\underline{\underline{\tau_{max_{CF}} = 12.5 \; MPa}}$$

CH. 5.4 ANGLE OF TWIST

DEF:

$$\phi = \frac{T\,L}{J\,G}$$

T — TORQUE APPLIED TO SECTION (N·m)

L — LENGTH (m)

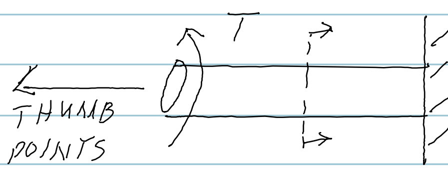
G — SHEAR MODULUS OF RIGIDITY (Pa)

J — POLAR MOMENT OF INERTIA (m^4)

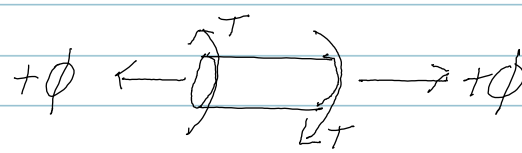
ϕ — ANGLE OF TWIST (radians)

DIRECTION OF TWIST

SIGN CONVENTION - RIGHT HAND RULE (RHR)



THUMB POINTS AWAY FROM THE END OF THE FBD = POSITIVE ϕ



$$\phi = \sum \frac{T_s\,L_s}{J_s\,G_s}$$

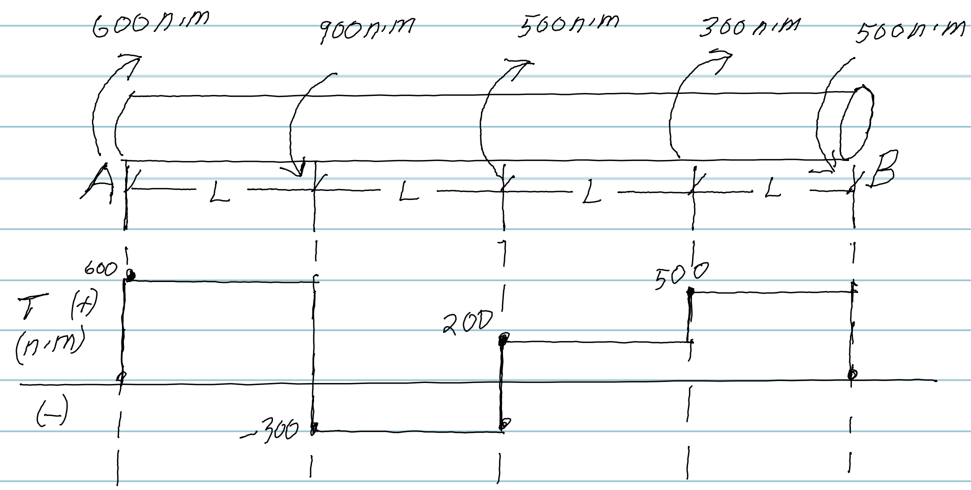
WHY IMPORTANT?

CH 5.4 PROBLEM F5-12

FIND: ANGLE OF TWIST BETWEEN ENDS OF SHAFT.
$G = 75\ GPa$, $dia = 40\ mm$, SOLID SHAFT, CONSTANT SPACING BETWEEN GEARS ($L = 200\ mm$).

SKETCH

$\phi_{A/B} = ?$



MISTAKES - COMMON UNIT

$d = 40mm = .04\ m$

$r = \frac{d}{2} = .02\ m$

$L = 200mm = 0.2\ m$

SOLUTION:

$$\phi = \sum \frac{T_s L_s}{J G} = \frac{L_s}{J_s G} \cdot \sum T_s$$

$$J = \frac{\pi}{2} r^4 = \frac{\pi}{2}(.02m)^4 = \frac{\pi}{2} \cdot \cancel{2.513} \cdot 1.6 \cdot 10^{-7} m^4 = 2.513 \cdot 10^{-7} m^4$$

$$\phi = \frac{.2\ m}{2.513 \cdot 10^{-7} m^4 \cdot (75 \cdot 10^9 Pa)} \cdot (+600 - 300 + 200 + 500)$$

$$\phi = 1.661 \cdot 10^{-5} \frac{m}{m^4 \cdot \frac{n}{m^2}} (1000\ n \cdot m) = .0106 \text{ radians}$$

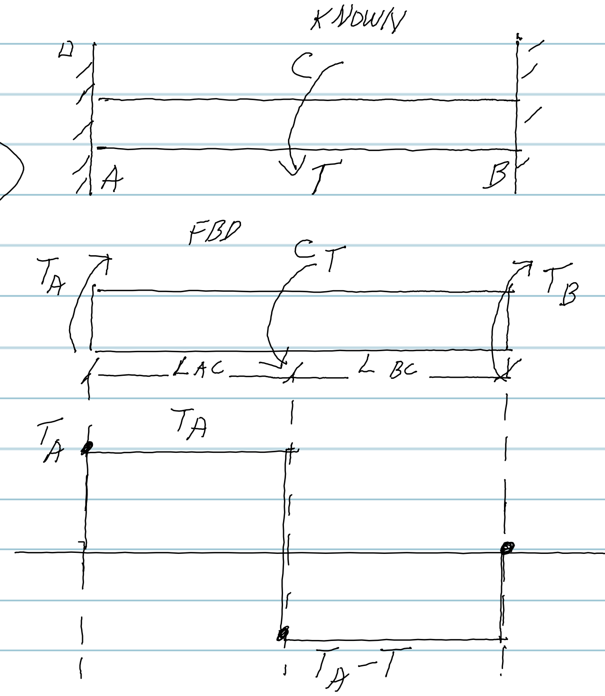
$$\phi = .0106 \text{ rad} \left(\frac{360\ ^\circ}{2\pi \text{ rad}}\right) = 0.608^\circ$$

CH 5.5 STATICALLY INDETERMINATE TORQUE LOADED MEMBERS

$$\Sigma M = \Sigma T = 0$$

① $T_A - T + T_B = 0$ 2 UNKNOWNS

SEE CH 4.4



① FBD

② COMPATABILITY EQ.

$$\phi_{A/B} = 0$$

$$\phi_{A/B} = \sum \frac{T_s L_s}{JG} = 0$$

$$\frac{T_{AC} L_{AC}}{JG} + \frac{(T_A - T) L_{BC}}{JG} = 0$$

$T_{AC} L_{AC} + (T_A - T) L_{BC} = 0$ SOLVE FOR $T_{AC} = ?$

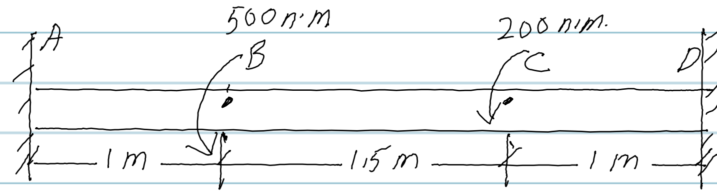
$T_A = T \dfrac{L_{BC}}{L}$, $T_B = T \dfrac{L_{AC}}{L}$ SIMPLE CASE.

CH. 5.5 PROBLEM 5-78

GIVEN:

A 992 STEEL SHAFT, SOLID

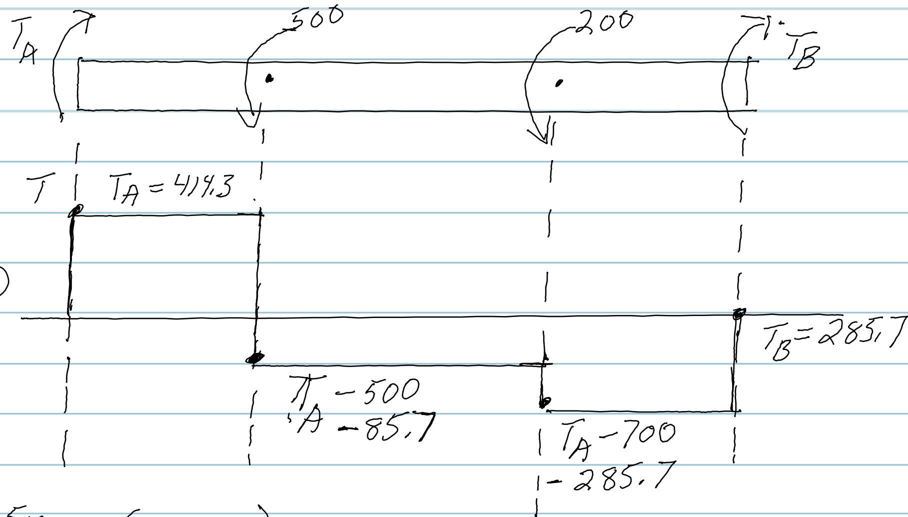
dia = 60 mm, FIXED END.



FIND: $\tau_{max} = ?$

$\Sigma M = \Sigma T = 0$

$+T_A - 500 - 200 + T_B = 0$ (1)



COMPATABILITY EQ.

$$\phi_{A/B} = \sum \frac{T_s L_s}{JG} = 0$$

$$\frac{T_A (1m)}{JG} + \frac{(T_A - 500)\,1.5m}{JG} + \frac{(T_A - 700)\,1m}{JG} = 0$$

$$T_A + 1.5T_A - 750 + T_A - 700 = 0 \Rightarrow 3.5T_A - 1450 = 0$$

$$T_A = 414.3 \text{ n·m}$$

FIND: $\tau_{max} = \frac{TC}{J} =$

$d = 60mm = .06 m$

$r = .03 m.$

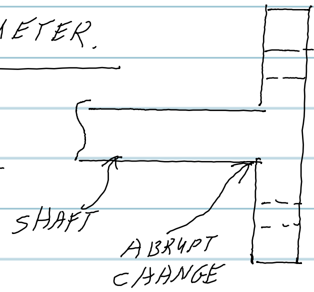
$$J = \frac{\pi}{2} r^4 = \frac{\pi}{2}(.03m)^4 = 1.272 \cdot 10^{-6} m^4$$

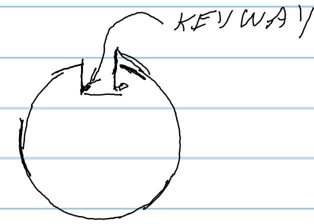
$$\tau_{max} = \frac{(414.3 \text{ n·m})\,.03m}{1.272 \cdot 10^{-6} m^4}$$

$$\tau_{max} = 9.77 \cdot 10^6 Pa = 9.77 \text{ MPa}$$

CH 5.8 STRESS CONCENTRATIONS IN SHAFTS

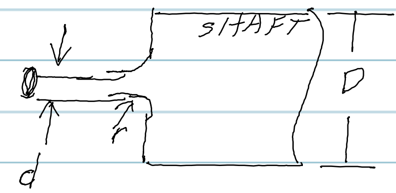
ABRUPT CHANGE IN DIAMETER.





USE TABLES – TESTING

$$\tau_{max} = K \frac{Tc}{J}$$

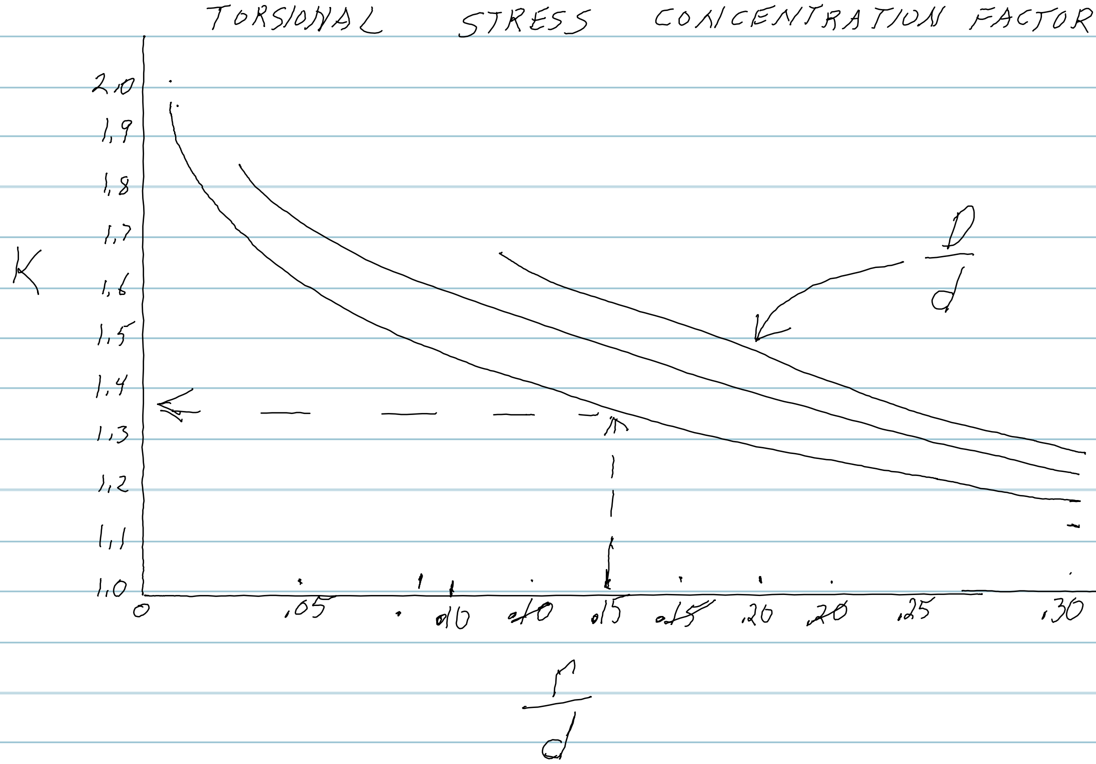


KEY ISSUES:

1) BRITTLE MATERIAL

2) CYCLIC TORSIONAL LOADING

TORSIONAL STRESS CONCENTRATION FACTOR



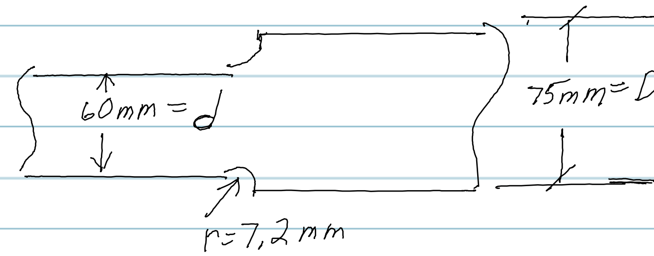
CH. 5.8 PROB. 5-122

GIVEN: TRANSITION TO SMALLER SHAFT DIAMETER

$\tau_{ALLOW} = \cancel{AAP}\ 55\ MPa$, RPM = 550, ~~r = 30 mm~~

SKETCH

FIND: $P_{MAX} = ?$



SOLUTION: $P = \dfrac{T \cdot RPM}{9550}$

$T_{max} = ?$

$\tau_{max} = K\dfrac{Tc}{J}$

$c = r = 30mm = .03\ m$

$J = \dfrac{\pi}{2} r^4 = \dfrac{\pi}{2}(.03)^4 = 1.272 \cdot 10^{-6}\ m^4$

$T = \dfrac{\tau_{max} \cdot J}{K \cdot r}$

FOR K:

$\dfrac{r}{d} = \dfrac{7.2mm}{60} = .12$

$\dfrac{D}{d} = \dfrac{75}{60} = 1.25$

$K = 1.3$

$T = \dfrac{(55 \cdot 10^9 Pa)\ 1.272 \cdot 10^{-6} m^4}{1.3\ (.03 m)}$

$T = 1794\ n \cdot m.$

POWER CALC.

$P = \dfrac{T \cdot RPM}{9550} = \dfrac{1794 \cdot 540}{9550}$

$\underline{\underline{P = 101\ KW}}$

$P = 101\ KW \left(\dfrac{1\ \ HP}{.746\ \ KW} \right) = \underline{\underline{136\ HP}}$