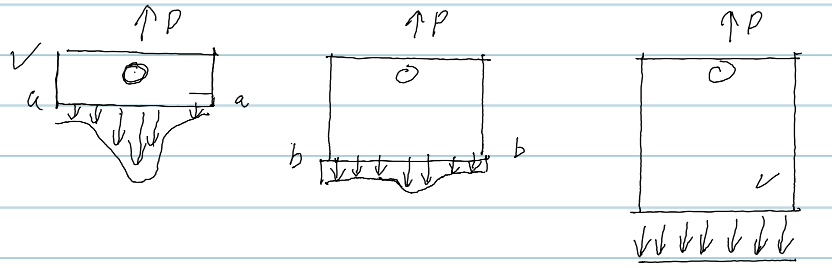
4.1 – 4.2 ELASTIC DEFORMATION ON AN AXIAL LOAD

SECTION 4.1 SAINT-VENANTS PRINCIPLE

STRESS AS YOU MOVE AWAY, σ EVENS OUT.



THINK OF ELASTIC MATERIAL FORCES SPREAD OUT & BECOME UNIFORM.

SECTION 4.2 ΔL (δ) FOR AXIAL LOADS

$$\sigma = E\epsilon$$

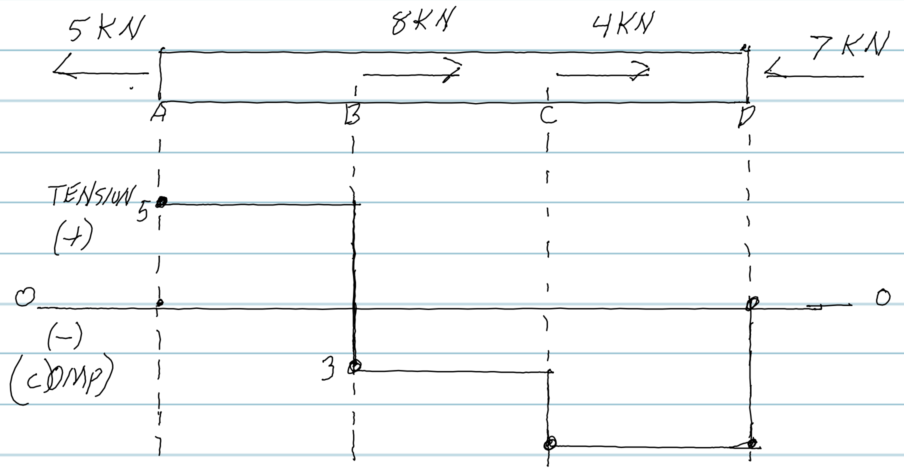
$$\frac{P}{A_0} = E\frac{\delta}{L_0}$$

$$\delta = \int_0^L \frac{P(x)\,dx}{A(x)\,E(x)}$$

$$\delta = \frac{PL_0}{AE}$$

AXIAL LOAD

HOW MUCH DOES ROD EXPAND OR CONTRACT DUE TO LOADS?



$$\delta_{A/D} = \sum \frac{PL_0}{AE} = \frac{L_0}{AE}\sum_{i=1}^{3} P_i$$

CH. 4.3 – 4.5 PRINCIPLE OF SUPERPOSITIONING

SECTION 4.3

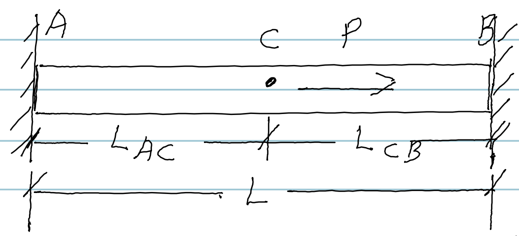
COVERED IN PHYSICS – LINEAR

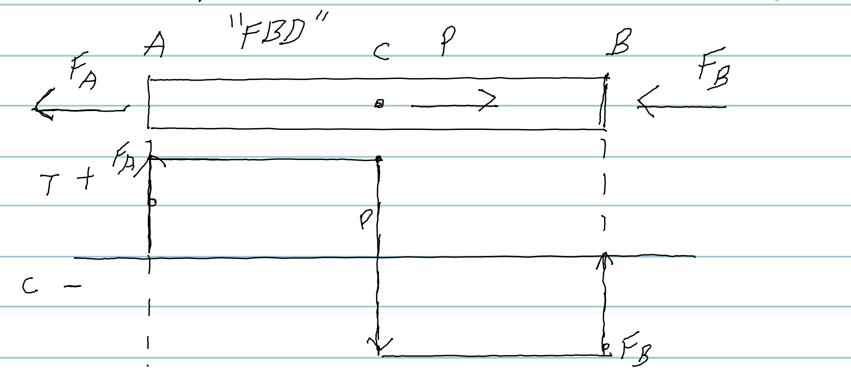
$\sigma = \frac{P}{A}$ $\sigma = E\epsilon$ $\delta = \frac{PL}{AE}$ LINEAR (ELASTIC RANGE ONLY)

KEY POINT: $\Delta L = \delta = \frac{PL}{AE}$ $\delta_T = \frac{P_1 L}{AE} + \frac{P_2 L}{AE} + \ldots$

TAKE AWAY: WE CAN ADD EFFECTS UP!

SECTION 4.4 SOLVING STATICALLY INDETERMINATE MEMBERS





SOL: $\Sigma F_x = 0 \Rightarrow$ ① $-F_A + P - F_B = 0$

$\delta_{AC} + \delta_{CB} = 0$ $\delta = \frac{PL}{AE}$

$\frac{(+F_A) L_{AC}}{AE} + \frac{(-F_B) L_{CB}}{AE} = 0 \Longrightarrow F_A L_{AC} - F_B L_{CB} = 0$ ②

SUB ① INTO ② $(P - F_B) L_{AC} - F_B L_{CB} = 0 \Rightarrow P L_{AC} - F_B L_{AC} - F_B L_{CB} = 0$

$P L_{AC} - F_B (L_{AC} + L_{CB}) = 0 \Rightarrow P L_{AC} = F_B (L) \Rightarrow$ * $F_B = \frac{P L_{AC}}{L}$

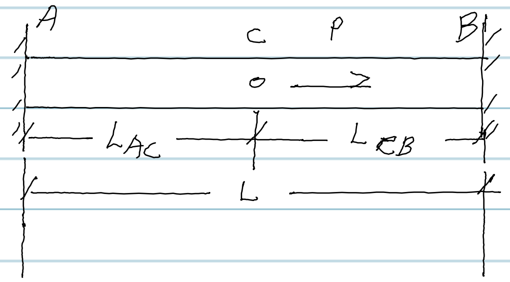
SUB: ANS INTO ① $F_A = P - F_B \Rightarrow F_A = P - \frac{P L_{AC}}{L} \Rightarrow F_A = P\left(1 - \frac{L_{AC}}{L}\right)$

$F_A \cdot L = L \cdot P\left[1 - \frac{L_{AC}}{L}\right] \Longrightarrow F_A \cdot L = P\left[L - L_{AC}\right] \Rightarrow F_A \cdot L = P \cdot L_{CB}$

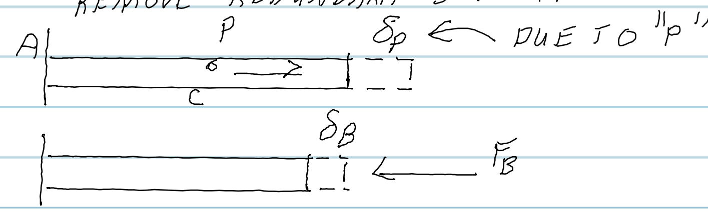
* $F_A = \frac{P L_{CB}}{L}$

CH. 4.3 – 4.5 SUPERPOSITION (CONT.)

SECTION 4.5 FORCE METHOD FOR AXIAL LOADED MEMBERS



REMOVE REDUNDANT SUPPORT



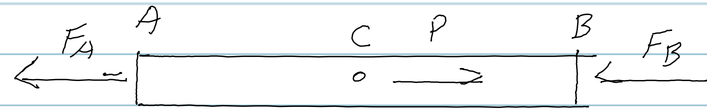
$$\delta_P = \frac{PL}{AE} = \frac{P L_{AC}}{AE} \qquad \delta_B = -\frac{F_B L}{AE} \qquad \delta_P + \delta_B = 0$$

$$\frac{P L_{AC}}{AE} - \frac{F_B L}{AE} = 0 \implies P L_{AC} - F_B L = 0$$

NEAT TRICK!

$$F_B L = P L_{AC} \implies F_B = \frac{P L_{AC}}{L}$$

GET F_A: DRAW FBD



$$-F_A + P - F_B = 0$$

$$F_A = P - F_B$$

$$F_A = P - \left[\frac{P L_{AC}}{L}\right] = P\left[1 - \frac{L_{AC}}{L}\right] = P\left[1 - \frac{(L - L_{CB})}{L}\right] = P\left(1 - 1 + \frac{L_{CB}}{L}\right)$$

$$F_A = \frac{P L_{CB}}{L}$$

TEMPERATURE CHANGE!!

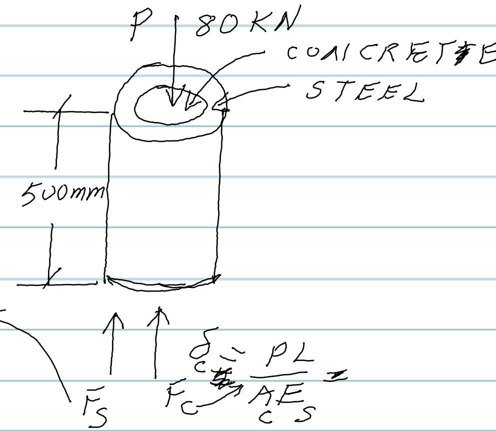
CH 4.3 – 4.5 PROB. 4–33

FIND: STRESS IN BOTH STEEL + CONCRETE FILLED PIPE.

SKETCH/GIVEN INFO.

$d_{OUTSIDE} = 80\,mm$ $E_s = 200\ GPa$

$d_{INSIDE} = 70\,mm$ $E_c = 24\ GPa$



$\delta_s = \dfrac{PL}{A_s E_s}$ $\delta_c = \dfrac{PL}{A_c E_c}$

SOLUTION:

1) FBD $\Sigma F_y = 0 \Rightarrow F_s + F_c - P = 0$ ①

2) CONTINUITY $\Rightarrow \delta_s = \delta_c \Rightarrow \dfrac{F_s L}{A_s E_s} = \dfrac{F_c L}{A_c E_c}$ ②

$$\frac{(P - F_c) L}{A_s E_s} = \frac{F_c L}{A_c E_c} \Rightarrow \frac{P}{A_s E_s} - \frac{F_c}{A_s E_s} = \frac{F_c}{A_c E_c}$$

$$\frac{F_c}{A_s E_s} + \frac{F_c}{A_c E_c} = \frac{P}{A_s E_s} \Rightarrow \frac{F_c (A_c E_c)}{A_s E_s A_c E_c} + \frac{F_c A_s E_s}{A_s E_s A_c E_c} = \frac{P}{A_s E_s}$$

$$F_c [A_c E_c + A_s E_s] = \frac{P [A_s E_s A_c E_c]}{A_s E_s}$$

$$F_c = \left[\frac{A_c E_c}{A_c E_c + A_s E_s}\right] P \qquad F_s = P - F_c$$

$$\sigma_s = \frac{F_s}{A_s} \qquad \sigma_c = \frac{F_c}{A_c}$$

MathCad P4-33_CONCRETE+STEEL_COLUMN.mcd
"PTC" FREE TO STUDENTS – FULL 30 days

I ADD SOLVE BLOCK IN MATHCAD.

CH. 4.6 THERMAL STRESS

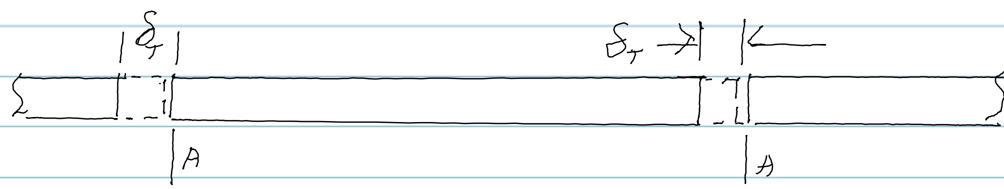
THERMAL COEF. (LINEAR)

$$\delta_T = \alpha \Delta T L_0 = \alpha (T_f - T_i) L_0$$

UNITS – °F, °C

PROB. 4-75

GIVEN: RAIL ROAD TRACK SECTION: A-36 STEEL, $L = 40'$, $A_s = 5.10 in^2$
$T_i = 90°F$, $T_f = 20°$ @ $T = 90°$ NO GAP



SOLUTION:

a) FIND $\delta_T = ?$ $\quad \delta_T = (6.6 \cdot 10^{-6} /°F)(20° - 90)(40')\left(\frac{12 in}{1 ft}\right) = \underline{\underline{0.348 in}}$

b) $T_i = 90°$ TOUCHING $\quad T_f = 110°$
$P = ?$

$$\delta_T = 6.6 \cdot 10^{-6}(110 - 90)(40')\left(\frac{12 in}{1}\right) = .0636 in$$

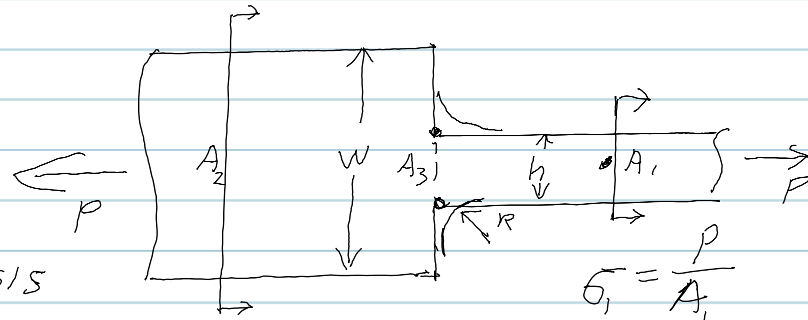
PUSH BACK

$$\delta = \frac{PL}{AE} \Rightarrow P = \frac{\delta AE}{L} = \frac{(.0636 in)(5.10 in^2)(29,000 ksi)}{(40')\left(\frac{12 in}{1 ft}\right)}$$

$\underline{\underline{\delta_T = 19.5 \text{ KIPS}}}$

CH. 4.7 STRESS CONCENTRATION FACTOR

$$K = \frac{\sigma_{max}}{\sigma_{avg}}$$



$$\sigma_1 = \frac{P}{A_1}$$

$$\sigma_2 = \frac{P}{A_2}$$

$$\sigma_{Avg3} = \frac{P}{A_3} = \sigma_1$$

$$\sigma_{max} = K\sigma_{avg}$$

FINITE ELEMENT ANALYSIS (FEA) SOFTWARE

TAKE AWAY:

$\sigma_1 = 30$ ksi (ALLOWABLE)

$K = 3$

$\sigma_{max} = 3 \cdot 30\text{ksi} = 90$ ksi