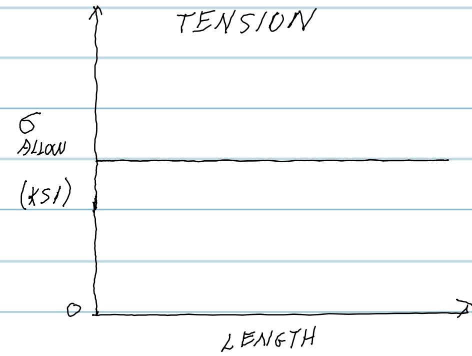
CH. 13 COLUMNS

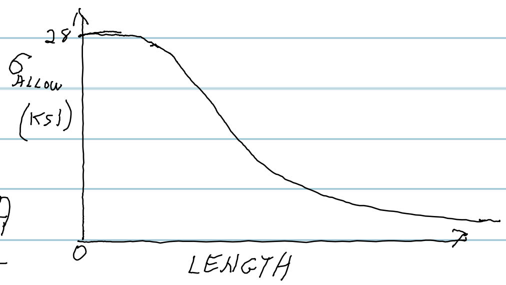
DEFINITION – COMPRESSION LOADS

IN TENSION

$$\sigma_{ALLOW} = \frac{P}{A}$$



IN COMPRESSION

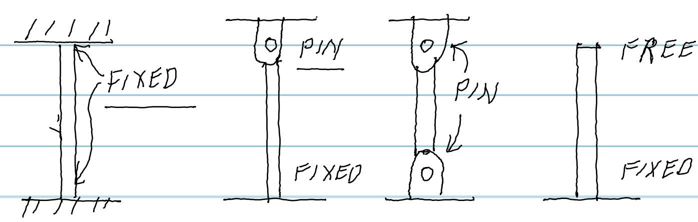


$$P_{ALLOW} = \sigma_{ALLOW} \cdot A$$

DEFINITIONS.

TYPE OF END COLUMN CONNECTIONS "K" (PINNED, FIXED, FREE)

K = 0.5 K = .7 K = 1 K = 2 (NO UNITS)



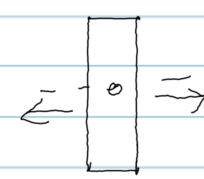
SLENDERNESS RATIO

$$\frac{KL}{r}$$

K ← CONNECTION TYPE

L ← LENGTH (EFFECTIVE)

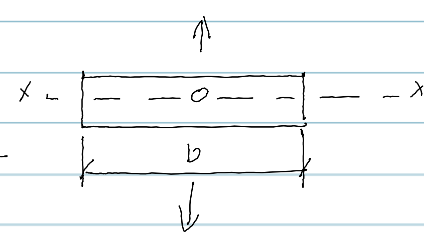
r ← RADIUS OF GYRATION



$$r = \sqrt{\frac{I}{A}}$$

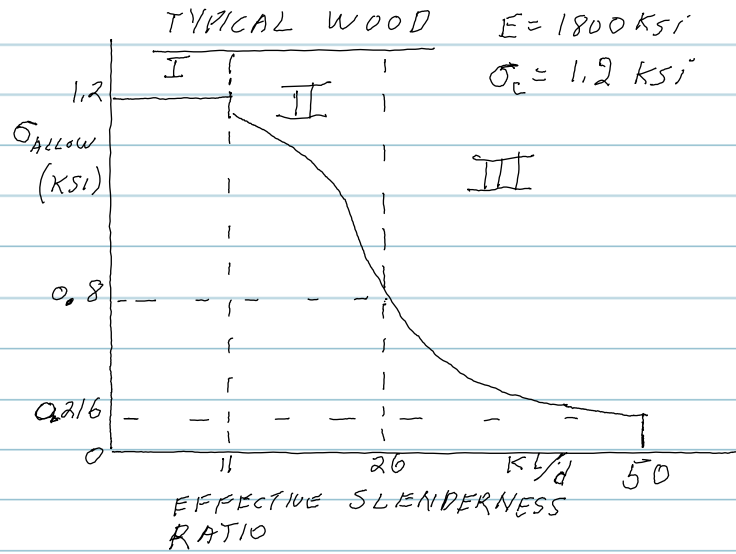
I ← CAN CHANGE – SMALLEST

$$I_{xx} = \frac{bh^3}{12}$$



CH. 13 COLUMNS (CONT.)

WOOD COLUMNS (13.6) (9th ed. 698)



ZONE I

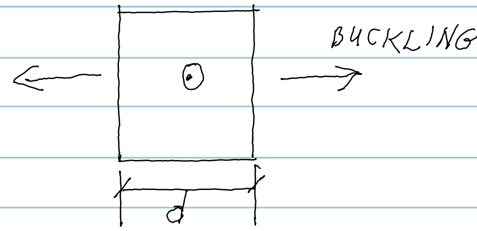
$$\sigma_{ALLOW} = 1.20 \text{ ksi} \qquad 0 < \frac{KL}{d} \le 11$$

ZONE II

$$\sigma_{ALLOW} = 1.20\left[1 - \frac{1}{3}\left(\frac{\frac{KL}{d}}{26}\right)^2\right]$$

FOR $11 < \frac{KL}{d} \le 26$

ZONE III

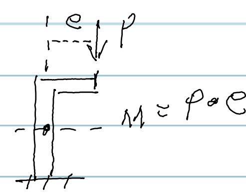


$$\sigma_{ALLOW} = \frac{540 \text{ ksi}}{\left(\frac{KL}{d}\right)^2} \qquad 26 < \frac{KL}{d} < 50$$

$$P_{ALLOW} = \sigma_{ALLOW} \cdot A$$

RESTRICTIONS

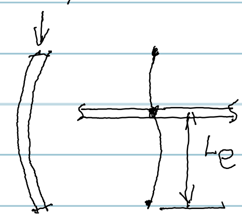
1) RECTANGULAR WOOD (SOLID) COLUMNS



2) LOAD APPLIED ON CENTROID OF CROSS SECTION

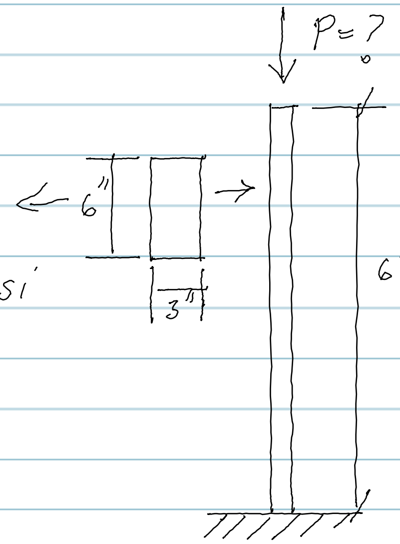
3) LENGTH "L" IS EFFECTIVE LENGTH OF COLUMN (UNBRACED)

4) FORMULA TAYLORED FOR TYPICAL WOOD (E + σ_c)



CH. 13 WOOD COLUMN DESIGN

PROB. 13-106



GIVEN 3x6 in WOOD COLUMN, L = 6 FT

FIND SAFE MAX. LOAD P = ?

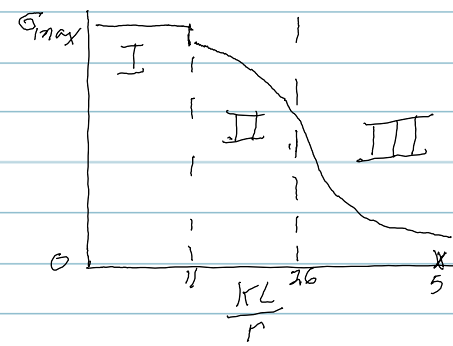
SOLUTION WOOD E = 1800 ksi; $\sigma_{max} = 1.2$ ksi

CONNECTION FACTOR K = 2,

SLENDERNESS RATIO

$$\frac{KL}{r} = \frac{2\cdot(6')\left(\frac{12\text{ in}}{1\text{ ft}}\right)}{3''} = 48$$

r ← LEAST DIMENSION



SELECT ZONE III

$$\sigma_{ALLOW} = \frac{540\text{ ksi}}{\left(\frac{KL}{r}\right)^2} = \frac{540\text{ ksi}}{48^2}$$

$$\sigma_{ALLOW} = 0.234\text{ ksi}$$

LOAD "P"

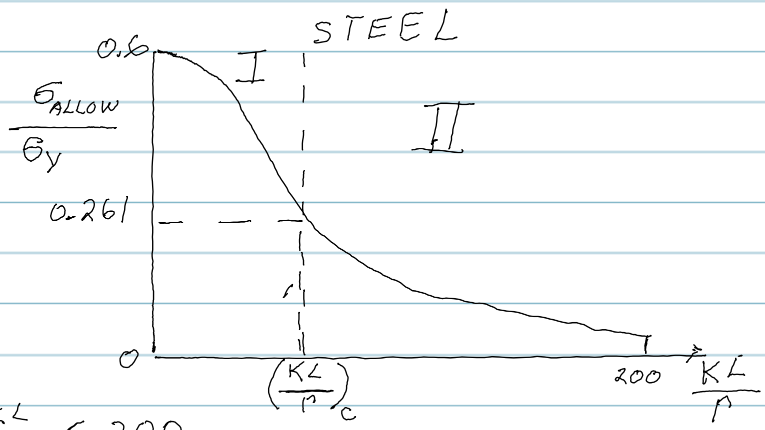
$$P = \sigma_{ALLOW}\cdot A = 0.234\text{ ksi}(3''\times 6'') = 4.2\text{ KIPS}$$

CH. 13.6 STEEL COLUMNS - ASD METHOD VS LRFD (Pg 18)

DESIGN

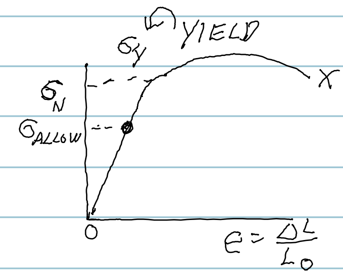
$$SL = \frac{KL}{r}$$

← RADIUS OF GYRATION $r = \sqrt{\frac{I}{A}}$



ZONE II

$$\sigma_{ALLOW} = \frac{12\pi^2 E}{23\left(\frac{KL}{r}\right)^2}, \quad \left(\frac{KL}{r}\right)_c \le \frac{KL}{r} \le 200$$



WHERE $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}}$

FACTOR OF SAFETY 1.67 TO 1.92

$$P = \sigma_{ALLOW} \cdot A$$

ZONE I

$$\sigma_{ALLOW} = \frac{\left[1 - \frac{\left(\frac{KL}{r}\right)^2}{2\left(\frac{KL}{r}\right)_c^2}\right]\sigma_y}{\left(\frac{5}{3}\right) + \left[\frac{\frac{3}{8}\left(\frac{KL}{r}\right)}{\left(\frac{KL}{r}\right)_c}\right] - \left[\frac{\left(\frac{KL}{r}\right)^3}{8\left(\frac{KL}{r}\right)_c^3}\right]}$$

(σ_y ← YIELD)

$$P = \sigma_{ALLOW} \cdot A$$

RESTRICTIONS

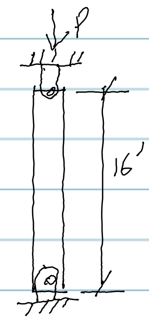
1) LOAD APPLIED ON CENTROID OF COLUMN

2) RADIUS OF GYRATION "r" DETERMINED FOR LEAST "I"

3) LENGTH "L" IS UNBRACED (EFFECTIVE LENGTH) OF COLUMN

CH 13.6 STEEL COLUMNS

EXAMPLE 13.6



GIVEN A992 STEEL W10X100, $L = 16\,FT$, AXIAL LOAD $P = ?$
PIN CONNECTIONS ON BOTH ENDS

FIND $P = ?$

SOLUTION

BEAM DETAILS $A = 29.4\,in^2$, $r_x = 4.6\,in$, $r_y = 2.65\,in$, $E = 29{,}000\,ksi$, $\sigma_y = 50\,ksi$

END CONNECTION FACTOR $K =$ ~~0.5~~ 1.0

SLENDERNESS RATIO $\frac{KL}{r} = \frac{1.0 \cdot 16\,FT}{2.65\,in}\left(\frac{12\,in}{1\,FT}\right) = 72.45$

CRITICAL SLENDERNESS RATIO $\left(\frac{KL}{r}\right)_C = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = 107$

SINCE $\frac{KL}{r} < \left(\frac{KL}{r}\right)_2$ $\quad 72.5 < 107$

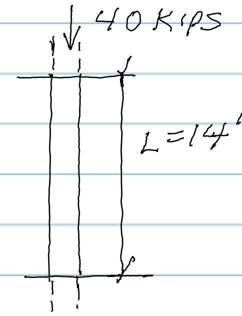
IN ZONE I

$\sigma_{ALLOW} = 20.48\,ksi$ NOT $23.93\,ksi$

$P_{ALLOW} = \sigma_{ALLOW} \cdot A = 20.48\,ksi \cdot 29.4\,in^2 = 602\,KIP$

PROB. 13-79

GIVEN A992 STEEL "W" SECTION, FIXED ENDS



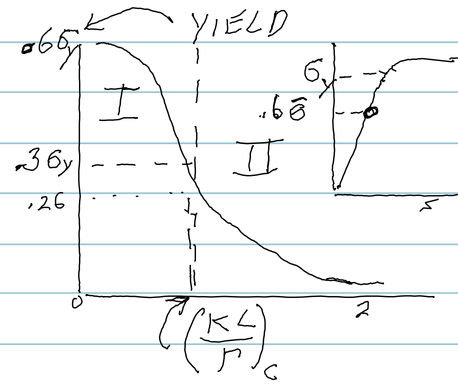
FIND "W" SECTION (LIGHTEST)

SOLUTION

SPECIFIC GIVENS $E = 29{,}000\ ksi$, $\sigma_y = 50\ ksi$, $L = 14'$, $K = 0.5$

$P_{req} = 40\ KIPS$

INITIAL SIZE GUESS



$$A_{GUESS} = \frac{P_{req}}{\sigma_{GUESS}} = \frac{40\ KIPS}{.3(50\ ksi)} = 2.7\ in^2$$

PICK A "W" SECTION w/ $A \cong A_{GUESS}$

W 6x9 HAS $A = 2.68\ in^2$ & LIGHT WEIGHT

$r_x = 2.47\ in$ & $r_y = 0.905\ in$

CALCULATE $$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2\ 29000\ ksi}{50\ ksi}} = 107.$$

ACTUAL $$\frac{KL}{r} = \frac{.5 \cdot 14 FT \left(\frac{12\ in}{FT}\right)}{.905\ in} = 92.8$$

SELECT A ZONE

$92.8 < 107$ ZONE I

$$\sigma_{ALLOW} = \frac{\left(1 - \frac{SR^2}{2 \cdot SR_c^2}\right)\sigma_y}{\frac{5}{3} + \frac{3}{8} \cdot \frac{SR}{SR_c} - \frac{SR^3}{8\ SR_c^3}}$$

$\sigma_{ALLOW} = 16.3\ ksi$

$P_{ALLOW} = A \cdot \sigma_{ALLOW} = 2.68\ in^2 \cdot 16.3\ ksi = 43.7\ ksi > 40\ KSI$ OK

CH 13.6 DESIGN OF ALUMINUM COLUMNS

ALUMINUM ASSOCIATION

COLUMN ALLOWABLE STRESSES

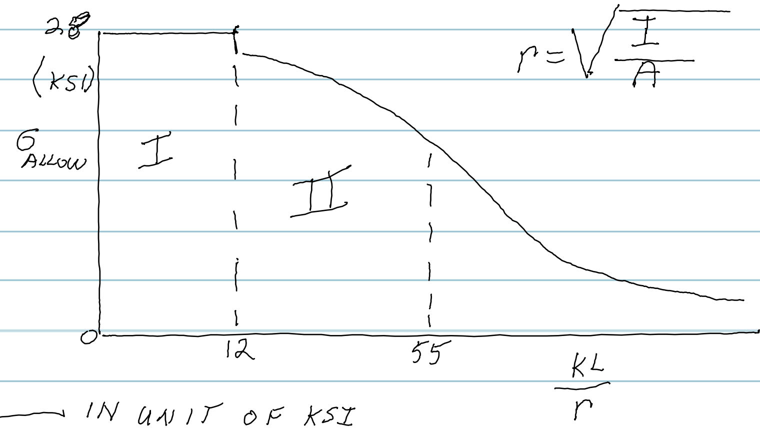
3 ZONES - UNIQUE SET OF FORMULAS FOR EACH TYPE
WE WILL USE 2014-T6 ALUMINUM ALLOY
COMMON IN BUILDING CONSTRUCTION

ZONE I

$\sigma_{ALLOW} = 28\,ksi \qquad 0 \le \frac{KL}{r} \le 12$

(r = RADIUS OF GYRATION)

$r = \sqrt{\frac{I}{A}}$



ZONE II

$\sigma_{ALLOW} = \left[30.7 - 0.23\left(\frac{KL}{r}\right)\right] \cdot ksi$ ← IN UNIT OF KSI (ksi)

$12 < \frac{KL}{r} < 55$

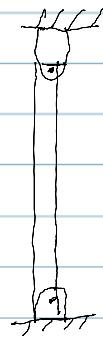
ZONE III

$\sigma_{ALLOW} = \frac{54000\,ksi}{\left(\frac{KL}{r}\right)^2} \qquad 55 \le \frac{KL}{r}$

CH 13.6 DESIGN OF ALUMINUM COLUMNS

PROB. 13-99

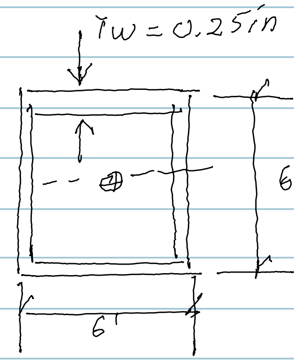
GIVEN 2014-T6 ALUMINUM ALLOY, SQUARE 6"x6"
WALL THICKNESS = 0.25 in, L = 10'



FIND MAX. SAFE LOAD P = ?

SOLUTION

FIND SLENDERNESS RATIO



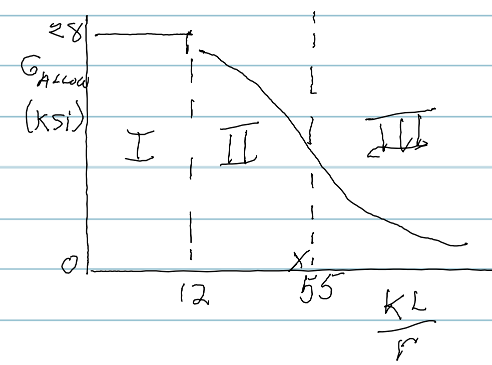
$$A_{cs} = (6")^2 - (5.5")^2 = 5.75\ in^2$$

$$I = \frac{1}{12}(6")(6")^3 - \frac{1}{12}\cdot 5.5\cdot 5.5^3$$

$$I = 31.75\ in^4 \qquad r = \sqrt{\frac{5.75\ in^2}{31.75\ in^4}} = 2.35\ in \quad \text{REVERSE}$$

$$\frac{KL}{r} = \frac{1\cdot 10'\left(\frac{12\ in}{1\ ft}\right)}{2.35\ in} = 51.1$$

ZONE II



$$\sigma_{ALLOW} = (30.7 - 0.23\,SR)\ ksi$$

$$\sigma_{ALLOW} = [30.7 - 0.23(51.1)]\ ksi$$

$$\sigma_{ALLOW} = 18.95\ ksi$$

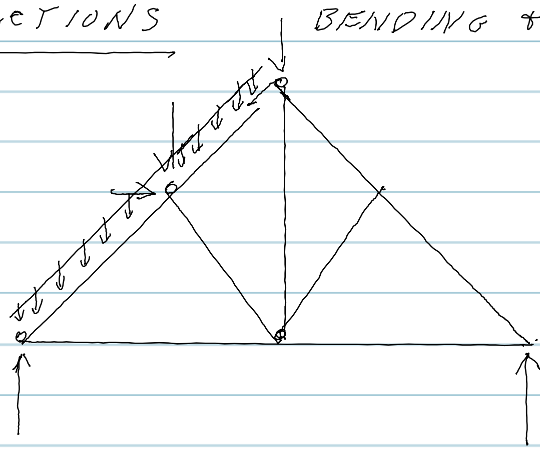
MAX. LOAD

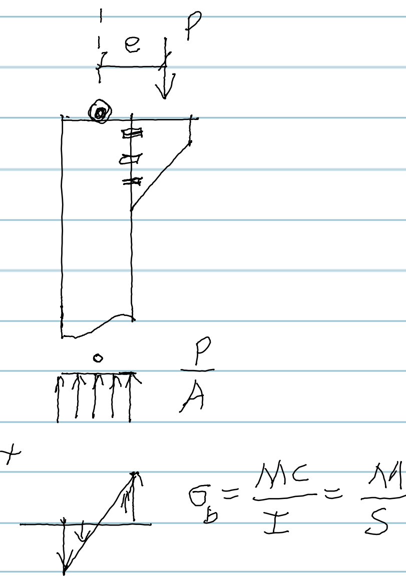
$$P_{ALLOW} = A_{cs}\cdot\sigma_{ALLOW} = 5.75\ in^2\cdot 18.95\ ksi = 109\ KIPS$$

CH. 13.7 ECCENTRIC LOADING OF COLUMNS

INTERACTIONS

BENDING & COMPRESSION IN COLUMNS





$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{P \cdot e \cdot c}{I}$$

MAKE SURE

$$\sigma_{max} \leq \sigma_{ALLOW} \{\text{COLUMN BY ITSELF}\}$$

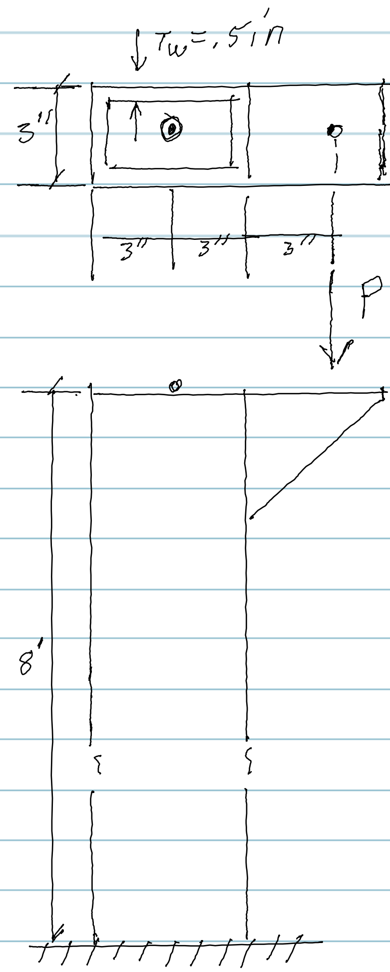
=

CH 13.7 ECCENTRIC LOADING ON COLUMN PROB. 13-119

GIVEN: 2014-T6 ALUMINUM HOLLOW COLUMN

USE ALLOWABLE STRESS METHOD

FIND MAX. LOAD $P = ?$



SOLUTION

FIRST - FIND COLUMN σ_{ALLOW}

$$A = 6 \times 3 - 5 \times 2 = 8 in^2$$

$$\square \quad I = \frac{1}{12}(6)3^3 - \frac{1}{12}(5)2^3 = 10.17 in^4$$

$$\square \quad r = \sqrt{\frac{I}{A}} = 1.127 in$$

SLENDERNESS RATIO $\frac{KL}{r} = \frac{2 \cdot 8' \cdot 12}{1.127} = 170.3$

ZONE III

$$\sigma_{ALLOW} = \frac{54000 \ ksi}{\left(\frac{KL}{r}\right)^2} = 1.862 \ ksi$$

COMBINE STRESSES $\sigma_{max} \leq \sigma_{allow}$

$$\sigma_{max} = \sigma_{allow} = \frac{P}{A} + \frac{Mc}{I_b} = \frac{P}{A} + \frac{P \cdot off \cdot c}{I_b}$$

where $c = 3 in$ { DISTANCE IN BENDING DIR. }

$off = 6 in$ { DISTANCE FROM CENTROID TO P. }

$$I_b = \frac{1}{12}(3)6^3 - \frac{1}{12}(2)5^3 = 33.2 in^4$$

$$\square \quad P = \frac{A \cdot \sigma_{ALLOW}}{\left(1 + \frac{off \cdot c \cdot A}{I_b}\right)}$$

$$P = \frac{8 in^2 \cdot 1.862 \ ksi}{\left(1 + \frac{6'' \cdot 3'' \cdot 8 in^2}{33.2 in^4}\right)} = 2.79 \ KIPS$$