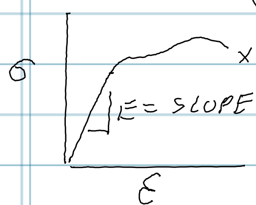
CH. 10.1 - 10.5 STRAIN EQUATIONS

10.1 STRAIN

$\sigma = E\,\varepsilon$ — σ: NORMAL STRESS, ε: STRAIN, E: MODULUS OF ELASTICITY

$\tau = G\,\gamma$ — τ: SHEAR STRESS, γ: SHEAR STRAIN, G: MODULUS OF REGIDITY



GET SIMILAR FORMULA'S

CH. 9 $\Rightarrow$ σ_x σ_y τ_{xy}

CH. 10 $\Rightarrow$ ε_x ε_y γ_{xy}

10.2 PRINCIPAL STRAINS

$$\tan(2\theta_p) = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

- $$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

- EQUATIONS 10-5 THRU 10-12

10.3 MOHR'S CIRCLE

COMPUTER GRAPH OF MOHR'S CIRCLE
(SEE MathCad file STRAIN TRANFORMATIONS.mcd)

- WILL NOT USE MOHR'S CIRCLE

10.4 ABSOLUTE MAXIMUM SHEAR STRAIN

- IF STRAIN ON PRINCIPAL AXIS HAVE THE SAME SIGN
  THEN $\gamma_{\substack{ABS\\MAX}}$ = LARGER OF ε_1 or ε_2

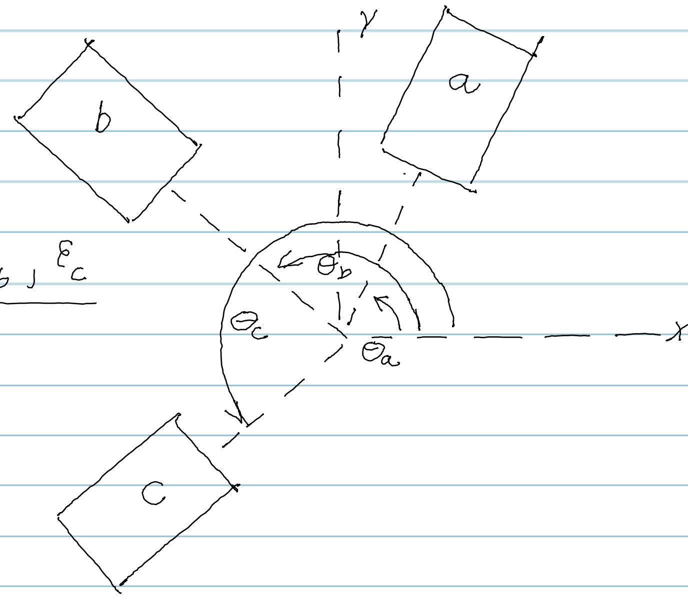
  IF STRAINS (PRINCIPAL AXIS) HAVE OPPOSITE SIGNS
  THEN $\gamma_{\substack{ABS\\MAX}} = \gamma_{\substack{MAX\\IN\text{-}PLANE}}$

10.5 STRAIN GAGE – CHANGE IN LENGTH CHANGES ELECTRICAL RESISTANCE

STRAIN ROSETTES

KNOWN ANGLES $\theta_a, \theta_b, \theta_c$

MEASURE STRAINS $\varepsilon_a, \varepsilon_b, \varepsilon_c$



☐ SUB. INTO:

$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy} \sin\theta_a \cos\theta_a$$

$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy} \sin(\theta_b)\cos(\theta_b)$$

$$\varepsilon_c = \varepsilon_x \cos^2(\theta_c) + \varepsilon_y \sin^2\theta_c + \gamma_{xy} \sin(\theta_c)\cos(\theta_c)$$

SOLUTION: 3 EQUATIONS FOR 3 UNKNOWN STRAINS – $\varepsilon_x, \varepsilon_y, \gamma_{xy}$

2 GEOMETRIC CASES SOLVED IN BOOK.

☐ USE COMPUTER PROGRAM FOR GENERAL CASE.
(SEE STRAIN TRANSFORMATIONS.mcd)

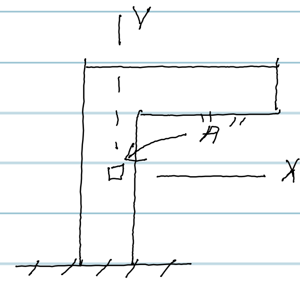
TRANSFORMATION RESULT: $\varepsilon_x, \varepsilon_y, \gamma_{xy}$

USE THESE VALUES @ INPUT IN STRAIN TRANSFORMATION EQUATIONS; TO FIND:

- STRAIN IN ANY DIRECTION ε_x' (or in program ε_{xp})
- PRINCIPAL AXIS STRAINS ε_1 & ε_2
- MAX. IN-PLANE SHEAR STRAIN $\gamma_{MAX\ IN\text{-}PLANE}$

CH. 10.1 – 10.5 PROB. 10-22 STRAIN TRANSFORMATION

GIVEN: STRAINS @ POINT "A"



$$\varepsilon_x = 300 \cdot 10^{-6} = 300\,\mu$$

$$\varepsilon_y = 550 \cdot 10^{-6} = 550\,\mu$$

$$\gamma_{xy} = -650 \cdot 10^{-6} = -650\,\mu$$

FIND: a) PRINCIPAL STRAINS $\varepsilon_1 = ?$ $\varepsilon_2 = ?$

b) $\gamma_{\substack{MAX\\IN\text{-}PLANE}} = ?$ c) $\gamma_{\substack{ABS\\MAX}} = ?$

SOLUTION

a)
$$\varepsilon_1 = \frac{\varepsilon_x + \varepsilon_y}{2} + \sqrt{\frac{\varepsilon_x + \varepsilon_y}{2} + \left(\frac{\gamma_{xy}}{2}\right)^2} \qquad \underline{\underline{\varepsilon_1 = 777\,\mu}}$$

$$\varepsilon_2 = \frac{\varepsilon_x + \varepsilon_y}{2} - \sqrt{\frac{\varepsilon_x + \varepsilon_y}{2} + \left(\frac{\gamma_{xy}}{2}\right)^2} \qquad \underline{\underline{\varepsilon_2 = 78.4\,\mu}}$$

b)
$$\gamma_{\substack{max\\IN\text{-}PLANE}} = 2\sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} \qquad \underline{\underline{\gamma_{\substack{MAX\\INPLANE}} = 698\,\mu}}$$

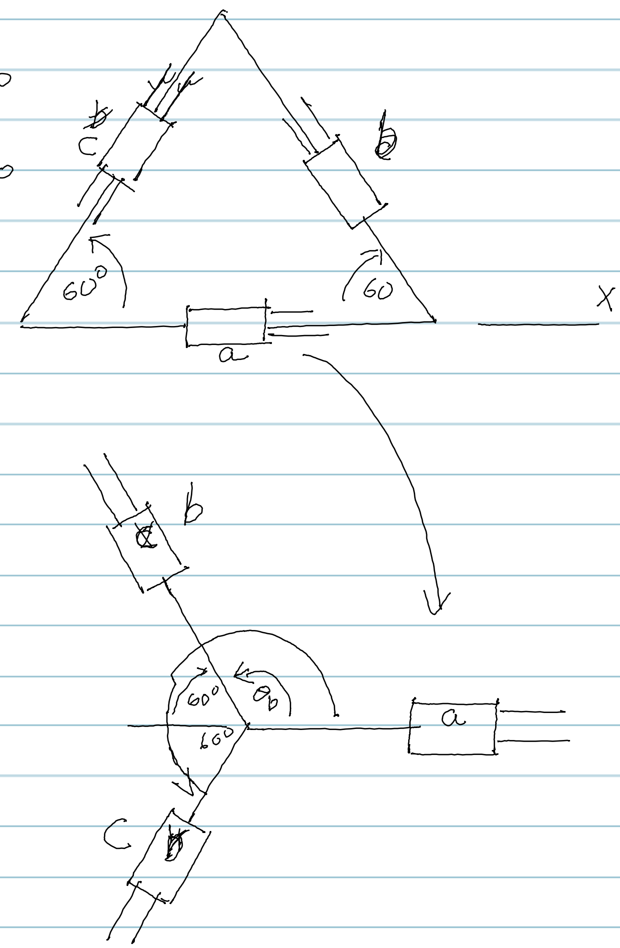
c) ε_1 & ε_2 HAVE SAME SIGN SO

*IMPORTANT: $\gamma_{\substack{ABS\\MAX}}$ = LARGER of ε_1 or ε_2

$$\gamma_{\substack{ABS\\MAX}} = \underline{\underline{777\,\mu}}$$

IF OPPOSITE SIGNS (ε_1 + ε_2) THEN $\gamma_{\substack{ABS\\MAX}} = \gamma_{\substack{max\\INPLANE}}$

CH. 10.1 – 10.5 PROB. 10-26 STRAIN ROSETTES



GIVEN:

$\varepsilon_a = 300\ \mu$ $\theta_a = 0°$

$\varepsilon_b = -150\ \mu$ $\theta_b = 120°$

$\varepsilon_c = -450\ \mu$ $\theta_c = 240°$

FIND! a) PRINCIPAL STRAINS

$\varepsilon_1 = ?\ \ \&\ \varepsilon_2 = ?$

$\theta_p = ?$

b) $\gamma_{\substack{max \\ \text{IN-PLANE}}} = ?$, $\theta_{\substack{MAX \\ \text{IN-PLANE}}} = ?$

SOLUTION

USE ROSETTE EQUATIONS

$\varepsilon_x = 300\mu$ $\varepsilon_y = -500\mu$

$\gamma_{xy} = -346.4\ \mu$

STRAIN TRANSFORMATIONS

a) PRINCIPAL AXIS $\theta = 11.7°$ $\varepsilon_1 = \varepsilon_{xp} = 336\mu$

$\varepsilon_2 = \varepsilon_{yp} = -536\ \mu$

b) $\gamma_{\substack{max \\ \text{IN-LINE}}} = 872\ \mu$ $\theta = -33.9°$

$\varepsilon_{AVG} = -100\mu$

CH. 11.1 – 11.2 PRISMATIC BEAM DESIGN

BEAMS – CARRY LOADS THRU BENDING

PRISMATIC BEAM – CONSTANT CROSS SECTION

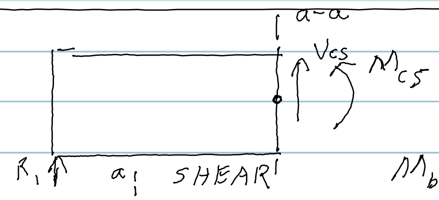
DESIGN OBJECTIVES

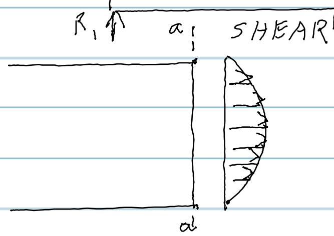
1) SAFETY
2) CUSTOMER OBJECTIVES
3) COST
4) DURABILITY

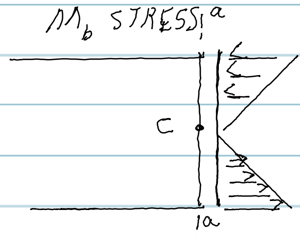
TYPES OF BEAM FAILURES

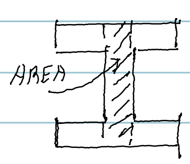
1) BENDING STRESS
2) SHEAR
3) TWISTING OUT OF ALIGNMENT
4) TORSIONAL LOADS (USUAL IN SHAFT DESIGN)
5) EXCESSIVE DEFLECTION
6) CONCENTRATED POINT LOADS (BEARING PLATES)
7) FAILURE @ CONNECTIONS (DETAILING)

REVIEW BENDING + SHEAR STRESS IN BEAMS

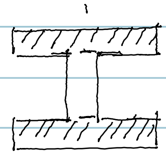


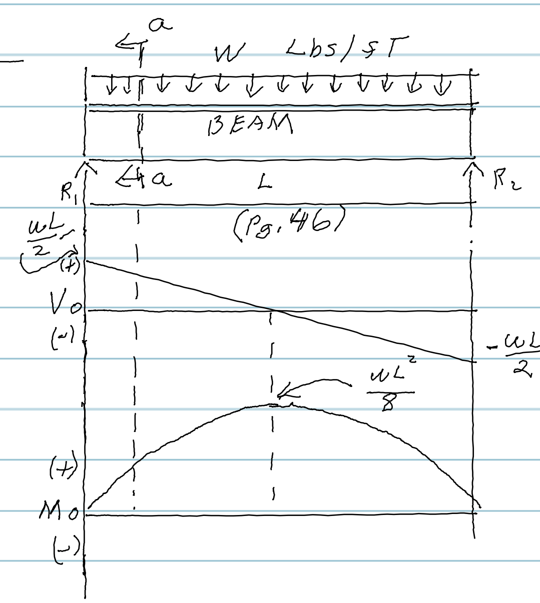






$$\tau = \frac{V}{A_{WEB}}$$





CH. 11.1 – 11.2 BEAM DESIGN (CONT.)

11.2 SECTION MODULUS

FLEXURE FORMULA

SECTION MODULUS (in^3)

$$\sigma_N = \sigma_b = \frac{M_b\, c}{I} \qquad S \equiv \frac{I}{c}$$

FROM DESIGN (Pg 47)

$$\sigma_b = \frac{M_b}{S}$$

STRENGTH OF SHAPE

BENDING STRESS AT EDGE (MAX @ C.S.)

MATERIAL PROPERTY

DESIGN PROCESS

FOR BENDING MOMENT

1) DRAW SHEAR + MOMENT DIAGRAM
2) SELECT LARGEST $|M_b|$ AS WORST CASE (M_{max})
3) CALCULATE REQUIRED MINIMUM "S_{MIN}" $\quad S_{MIN} = \dfrac{M_{max}}{\sigma_{ALLOW}}$
4) SELECT LIGHTEST BEAM WITH $S > S_{MIN}$ (A. 'B')

CHECK FOR SHEAR FAILURE

SHEAR SHORTCUTS – RECTANGLE C.S'S $\quad \tau_{CENT.} = 1.5\,\dfrac{V_{max}}{A_{CS}}$

CIRCULAR CS'S $\quad \tau_{CENT.} = 1.33\,\dfrac{V_{max}}{A_{CS}}$

"I" + "W" BEAMS $\quad \tau \cong \dfrac{V_{max}}{A_{WEB}} \qquad \tau < \tau_{ALLOW}$

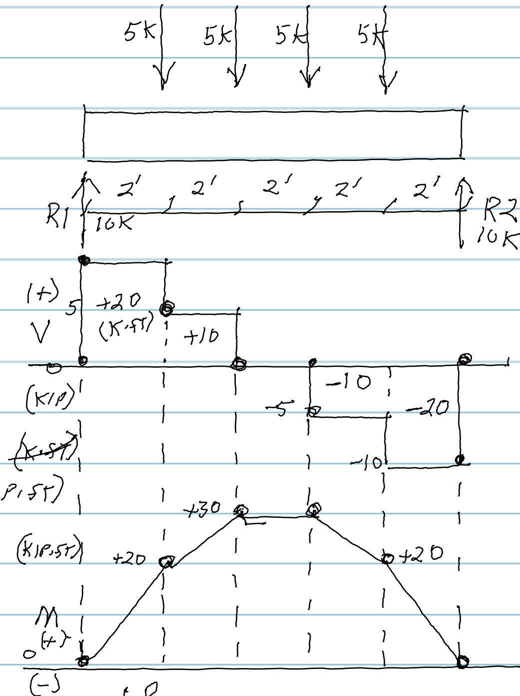
CHECK FOR DEFLECTION

CH. 11.1 – 11.2 PROB. 11-5

GIVEN: $\sigma_{ALLOW} = 24\ ksi$

$\tau_{ALLOW} = 14\ ksi$

FIND: BEAM SIZE



SOLUTION

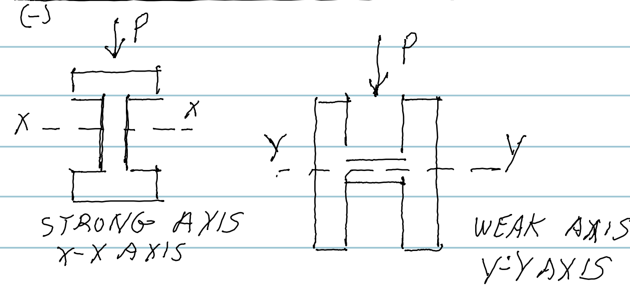
FLEXURE FORMULA $\sigma_b = \dfrac{M}{S} \Rightarrow S_{MIN} = \dfrac{M_{max}}{\sigma_{ALLOW}}$

$M_{max} = +30 (KIP \cdot FT)$

$$S_{min} = \frac{+30\ KIP \cdot FT}{24\ KSI}\left(\frac{12\ in}{1\ FT}\right) = 15\ in^3$$

SELECT BEAM S_{MIN} OR LARGER

"S_{XX}" AXIS OR "S_{YY}" AXIS



		$S_{XX} (in^3)$
☐	W 6 x 25	16.7
☐	W 8 x 24	20.9
☐	W 10 x 19	18.8
PICK ⇒ ☐	W 12 x 16	17.1
☐	W 14 x 22	29

$d = 11.99\ in,\ t_w = 0.22\ in$

CK FOR SHEAR STRESS

$$\tau_{ACT} = \frac{V_{max}}{A_{WEB}} = \frac{10\ KIPS}{(11.99\ in)(0.22\ in)} = 3.79\ ksi$$

CRITERIA $\tau_{ACT} < \tau_{ALLOW}$

$3.79\ ksi < 14\ ksi$ ✓ OK IN SHEAR

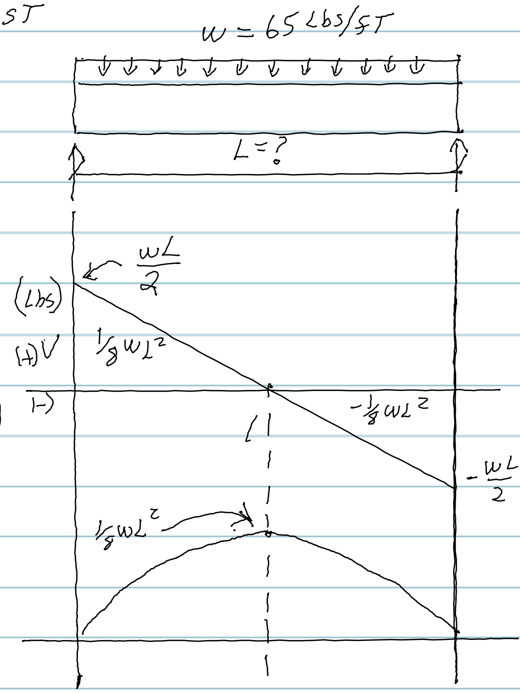
CK M_b CAPACITY USING W12x16

$$M_{ALLOW} = S_{ACT} \cdot \sigma_{ALLOW} = 17.1\ in^3 \cdot 24\ ksi = 410.4\ kip \cdot in\left(\frac{1\ FT}{12\ in}\right) = 34.2\ KIP \cdot FT$$

M_{req} (FROM S.M) $= 30\ kip \cdot FT < 34.2\ KIP \cdot FT$ OK! CK

CH. 11.1 – 11.2 SPAN LENGTH PROBLEM

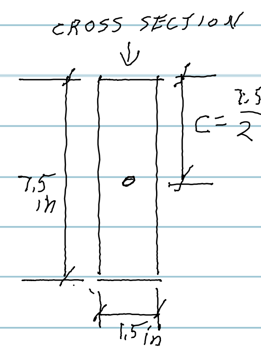
GIVEN: A 2x8 SOUTHERN-PINE FLOOR JOIST IS SUPPORTING A PLYWOOD SUBFLOOR THAT HAS AN ESTIMATED WEIGHT OF 50 Lbs/FT² (CODE). THE JOISTS ARE 16" ON-CENTER (OC - SPACED APART). THE JOIST CARRIES A UNIFORM LOAD OF 65 Lbs/FT. THE MATERIAL PROPERTIES OF THE PINE IS: $\sigma_{ALLOW\,b} = 1300\ psi$ & SHEAR STRESS (CALLED HORIZONTAL SHEAR STRESS) IS $\tau = 90\ psi$. THE JOIST IS SIMPLY SUPPORTED.



THE LARGEST SPAN "L" (SAFE)

SOLUTION: WORK BACKWARDS FROM ANSWER!

$$L = ? \Rightarrow M_b = \frac{1}{8} wL^2 \Rightarrow M_b = ?$$



FLEXURE FORMULA $\sigma_{ALLOW} = \frac{M_b}{S} \Rightarrow S = ? \Rightarrow S = \frac{I}{C} \Rightarrow I = ?$

$$I = \frac{bh^3}{12}$$

$$I = \frac{(1.5")(7.5\,in)^3}{12} = 52.7\ in^4$$

$$S = \frac{I}{c} = \frac{52.7\,in^4}{(7.5"/2)} = 14.06\ in^3$$

$$M_b = \sigma_{ALLOW} \cdot S = 1300\,psi \cdot 14.06\,in^3 = 1.688 \cdot 10^4\ in \cdot Lbs \left(\frac{1\,FT}{12\,in}\right) = 1406\ FT \cdot Lbs$$

$$L = \sqrt{\frac{8 \cdot M_b}{W}} = \sqrt{\frac{8 \cdot 1406\,FT \cdot Lbs}{65\,Lbs/FT}} = \underline{13.2\ FT}$$

CK HORIZONTAL SHEAR STRESS $\tau_{ACTUAL} = \frac{V_{max}}{A_{cs}} = \frac{\frac{1}{2} wL}{(7.5" \cdot 1.5")} = \frac{\frac{1}{2}(65\frac{Lb}{FT})\,13.2\,FT}{7.5" \cdot 1.5"}$

$\tau_{ACT} = 38\,psi < \tau_{ALLOW} = 90\,psi$ **OK IN SHEAR**

NOTE: DID NOT CONSIDER DEFLECTION.