ENGR-225 DESIGN OF MATERIAL STRUCTURES

INTRODUCTION –

CH 1.1 – 1.2 INTRODUCTION

BREAKING THINGS

STATICS – REVIEW FBD

TERMS – STATICS – 2 D – (FBD)

$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma M = 0 \Rightarrow \Sigma r \times F = 0$

2-D ⇒ COPLANNER FORCE SYSTEM

$\Sigma r_\perp \cdot F = 0$

$\Sigma M + \Sigma r_\perp \cdot F = 0$

3-D STATICS

$\Sigma F_x = 0 \quad \Sigma M_x = 0$

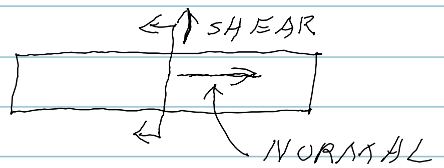
$\Sigma F_y = 0 \quad \Sigma M_y = 0$

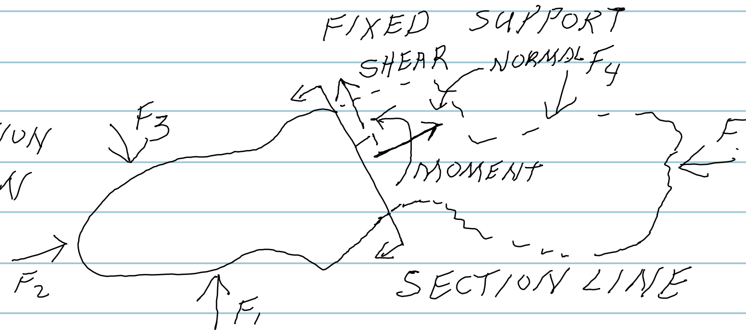
$\Sigma F_z = 0 \quad \Sigma M_z = 0$

BRAND NEW TERMS:

NORMAL FORCE ⇒ ⊥ SECTION

SHEAR FORCE ⇒ // SECTION





USUALLY – SECTION ⊥ TO LONG AXIS

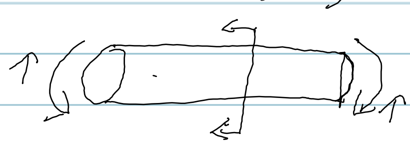
NORMAL USUALLY // LONG AXIS.

3-D – NORMAL ⊥ SECTION, SHEAR // TO SECTION

MOMENT IS IN THE PLANE OF THS SECTION.

SOMEWHERE IN Y-Y AXIS

TORSIONAL MOMENT (3-D) ⇒ TWISTING



RIGHT HAND RULE

RHR

TORQUE ⊥ SECTION

CH 1.1 – 1.2 (CONT.)

ADITIONAL TERMS ON FBD

FAMILIAR	NEW	EXAMPLES
FORCE @ DIST.	BODY FORCES	GRAVITY, EM FORCE
CONTACT	SURFACE FORCE ←	CONCENTRATED FORCE, DISTRIBUTED FORCE

INTERNAL FORCE – GOOD NEWS!!

DO SAME FBD'S AS IN STATICS.

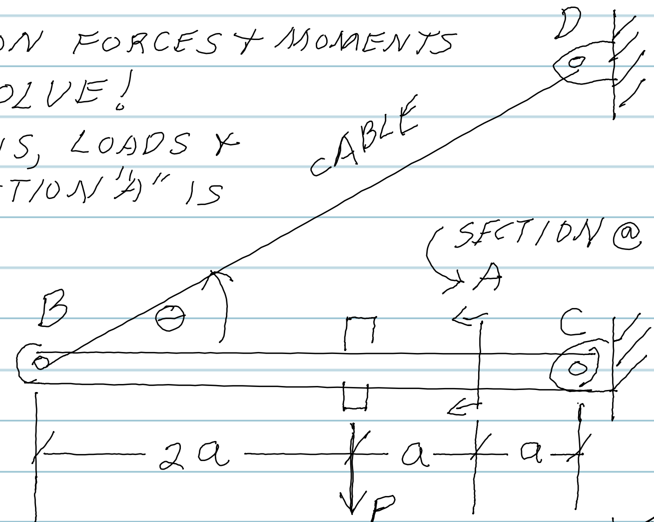
CUT THROUGH SECTION OF INTREST

RULES FOR FINDING INTERNAL FORCES:

1) USUALLY DO – EXTERNAL FBD → REACTIONS AT SUPPORTS
2) NEW FBD W/ CUT ACROSS SECTION OF INTREST.
3) LABEL ON FBD – NORMAL, SHEAR, + MOMENT – AT CENTROID OF SECTION CUT.
4) ESTABLISH X, Y, Z ORGIN @ CENTROID
5) VIP!! TAKE $\Sigma M_{CENTROID} = 0$ ⇐ BOTH NORMAL + SHEAR – PASS THROUGH CENTROID – BOTH EQUAL + 1 EQUATION + 1 UNK.
6) $\Sigma F_x = 0$, $\Sigma F_y = 0$
7) REMINDER ⇒ NEGATIVE RESULTANT REVERSES DIRECTION ON FBD – STILL CORRECT

CH 1.1 – 1.2 (CONT.)

FIND THE INTERNAL SECTION FORCES + MOMENTS @ SECTION "A". DO NOT SOLVE! ASSUME LETTERED DIMENSIONS, LOADS + ANGLES ARE KNOWN + SECTION "A" IS VERTICAL.



DRAW NECESSARY FBD'S + SHOW SOLUTION STEPS WITH RELEVANT EQUATIONS

1) STEP 1 – EXTERNAL – SOLVE

b) SOLVE $\Sigma M = \Sigma F \cdot r_{\perp} = 0$

$+P \cdot 2a \quad - F_D \cdot [4a \sin\theta] = 0$

$\sin\theta = \frac{r_{\perp}}{4a} \qquad \therefore F_D \Rightarrow$ KNOWN

c) $\Sigma F_x = 0$

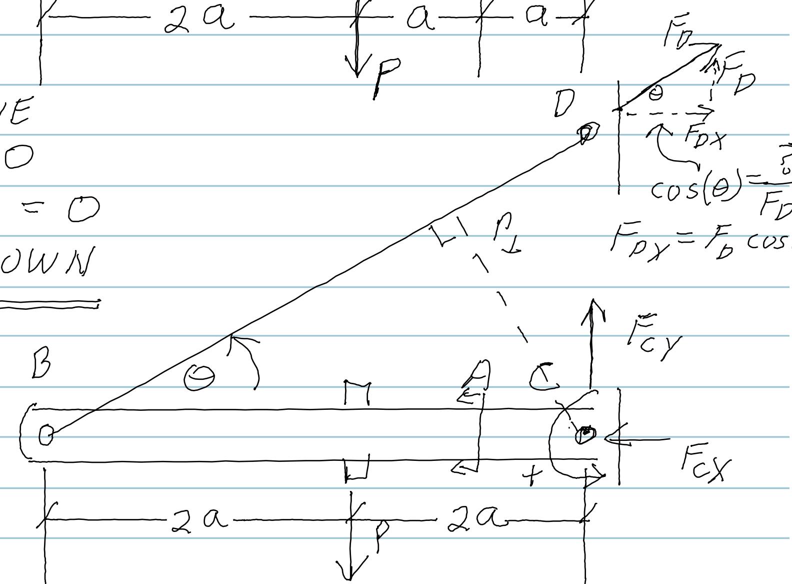
$-F_{cx} + F_D \cdot \cos\theta = 0$

F_{cx} KNOWN

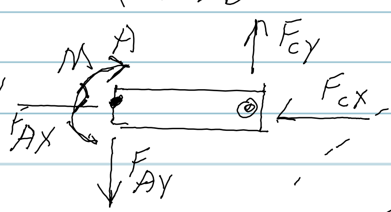
d) $\Sigma F_y = 0$

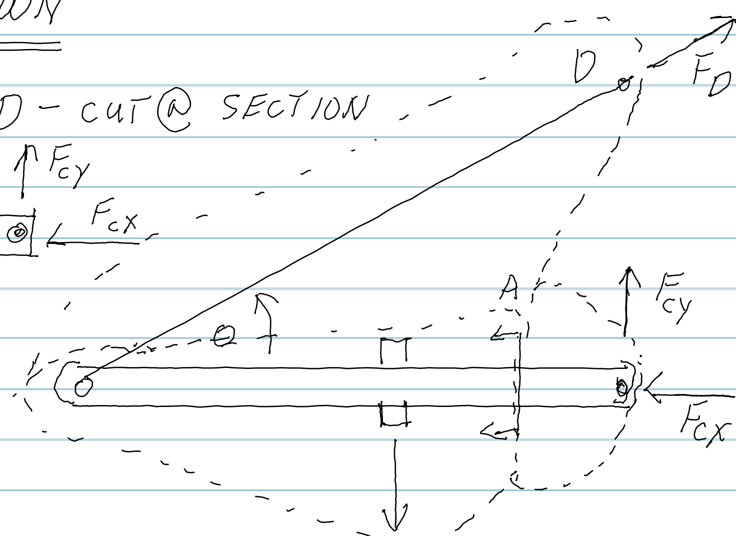
$-P + F_{cy} + F_{Dy} = 0$

$F_{Dy} = F_D \sin\theta \Rightarrow F_{cy}$ KNOWN



2) STEP 2 – ANOTHER FBD – CUT @ SECTION





$\Sigma F_x = 0 \Rightarrow F_{AX}$ KNOWN

$\Sigma F_y = 0 \Rightarrow F_{AY}$ KNOWN

$\Sigma M_A = 0$

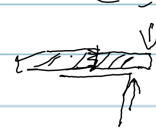
$\Sigma F \cdot r_{\perp} = 0$

$-M + F_{cy}(a) = 0$

M IS KNOWN

TAKE AWAY – $F_{AX} \Leftrightarrow$ NORMAL FORCE

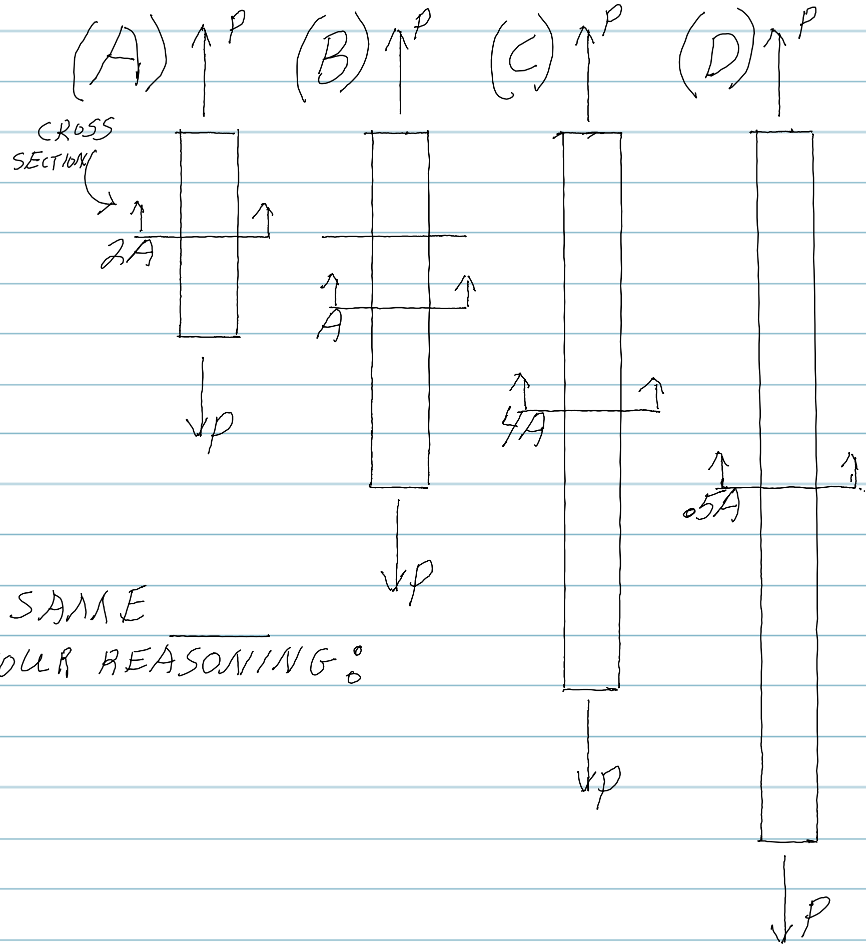
$F_{AY} \Leftrightarrow$ SHEAR FORCE



$M \Leftrightarrow M = F \cdot r_{\perp}$

☐ RANKING TASK 1

CONSIDER THE 4 WOOD MEMBERS LABELED A-D WITH APPLIED FORCES AS SHOWN.
RANK THE NORMAL FORCES THAT EXIST AT THE CROSS SECTIONS SHOWN FROM GREATEST TO LEAST.
NOTE THE RELATIVE AREA OF EACH CROSS SECTION IS SHOWN BY EACH FIGURE (2A, A, 4A, .5A)



GREATEST
1) ____
2) ____
3) ____
4) ____
LEAST
ALL THE SAME ____
EXPLAIN YOUR REASONING:

CONFIDANT IN YOUR ANSWERS (CIRCLE ONE)
LOW FAIRLY POSITIVE

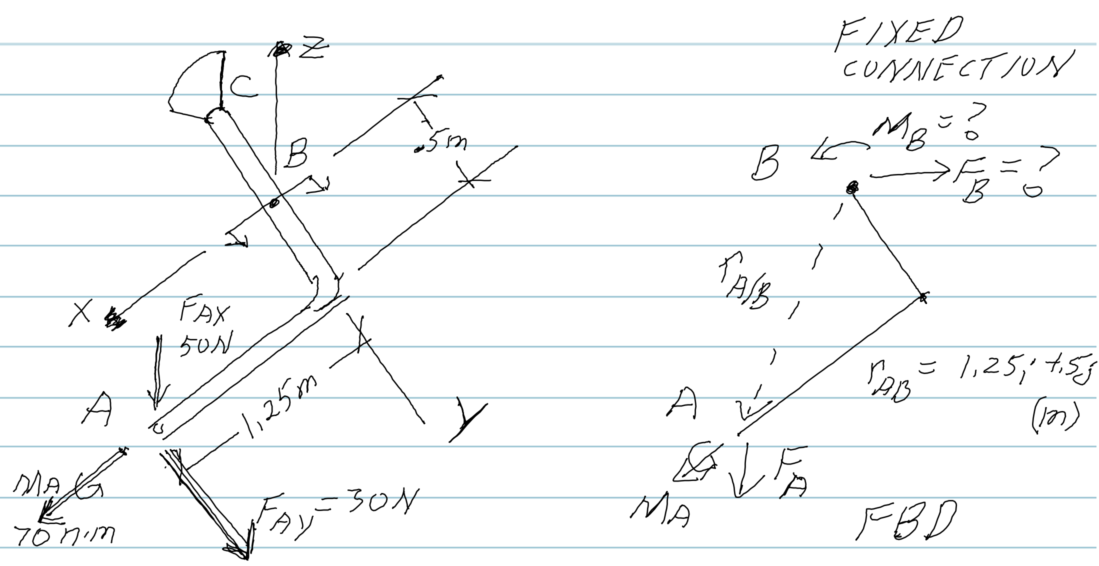
EXAMPLE PROBLEM 1.4

NOTE: BOOK USES COMPONENT FORM – WE WILL USE "X"

GIVEN / FIND: CALCULATE INTERNAL LOADINGS @ CROSS SECTION "B" ON THE PIPE.

$F_A = 0i + 30j - 50k$ (N), $M_A = +70i + 0j + 0k$ (n·m)

SKETCH



SOLUTION

1) $\Sigma F = 0$ $F_A + F_B = 0$ $F_B = 0i - 30j + 50k$ (N)

2) $\Sigma M + \Sigma r \times F = 0 \Rightarrow M_A + M_B + r_{A/B} \times F_A = 0$

$$r_{A/B} \times F = \begin{vmatrix} i & j & k \\ 1.25 & .5 & 0 \\ 0 & +30 & -50 \end{vmatrix} \begin{matrix} i & j \\ 1.25 & .5 \\ 0 & 30 \end{matrix} = [-25i + 37.5k] - [-62.5j] = -25i + 62.5j + 37.5k \text{ (n·m)}$$

$[+70i + 0i + 0k] + M_B + [-25i + 62.5j + 37.5k] = 0$

$M_B + 45i + 62.5j + 37.5k = 0$

$M_B = -45i - 62.5j - 37.5k$ (n·m)

FIND: NORMAL FORCE (N_B) BENDING MOMENT (MB_B)
SHEAR FORCE (V_B) TORSIONAL MOMENT (T_B)

EXAMPLE PROB. 1.4 (CONT.)

1) NORMAL FORCE – ⊥ TO CROSS SECTION (PLANE X-Z)

$F_B = 0 - 30j + 50k$ (N)

$N_B = F_{BY} = -30$ N (NEGATIVE SIGN – F_{BY})

$N_B = 30$ N (T)

2) SHEAR FORCE – // TO CROSS SECTION (PLANE X-Z)

$F_B = 0i - 30j + 50k$ (N)

$V_B = 0i + 50k$ (N) $= \sqrt{0^2 + 50^2} = 50$ N

3) TORSIONAL MOMENT – MOMENT COMPONENT ⊥ PLANE OF THE SECTION. (PLANE X-Z)

$M_B = -45i - 62.5j - 37.5k$ (N·m)

$T_B = -62.5$ N·m

4) BENDING MOMENT – MOMENT COMPONENTS IN PLANE

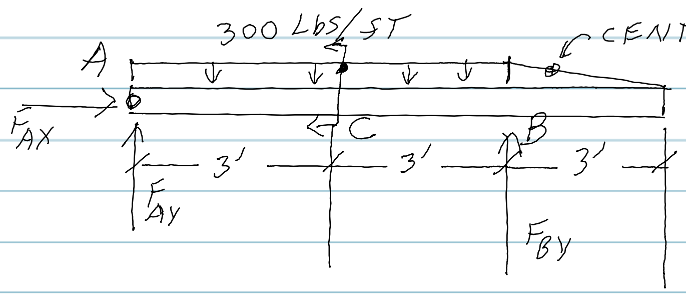
$M_B = -45i - 62.5j - 37.5k$ (N·m)

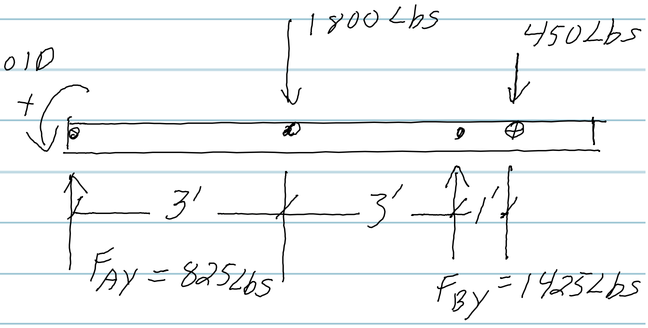
$MB_B = -45i + 0j - 37.5k = \sqrt{45^2 + 37.5^2} = 58.6$ (N·m)

PROBLEM F 1-5 (FUNDAMENTAL PROBLEM)

FIND: DETERMINE THE INTERNAL LOADING @ CROSS SECTION "C"

SKETCH





SOLUTION

STEP 1) SOLVE FOR REACTION - USE FBD

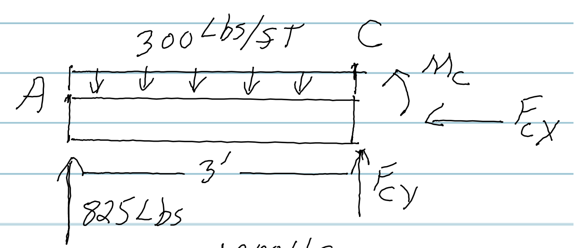
$\Sigma M = 0 \qquad +F_{AY} \cdot 0 - 1800 \cdot 3 + F_{BY} \cdot 6 - 450(7) = 0$

$-5400 + 6F_{BY} - 3150 = 0 \qquad F_{BY} = 1425 Lbs.$

$\Sigma F_y = 0$

$F_{AY} - 1800 + 1425 - 450 = 0 \Rightarrow F_{AY} = 825 Lbs$

STEP 2) REDRAW FBD W/ CUT @ "C"



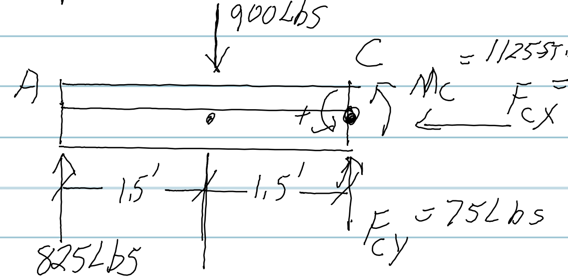
$\Sigma M_C = 0 \Rightarrow \Sigma M + \Sigma F \cdot r_\perp = 0$

$-(825 Lbs)3' + 900(1.5) + M_C = 0$

$-2475 + 1358 + M_C = 0$

$-1125 + M_C = 0$

$M_C = 1125\ FT \cdot Lbs$



$\Sigma F_y = 0$

$+825 - 900 + F_{cy} = 0 \qquad F_{cy} = 75 Lbs$

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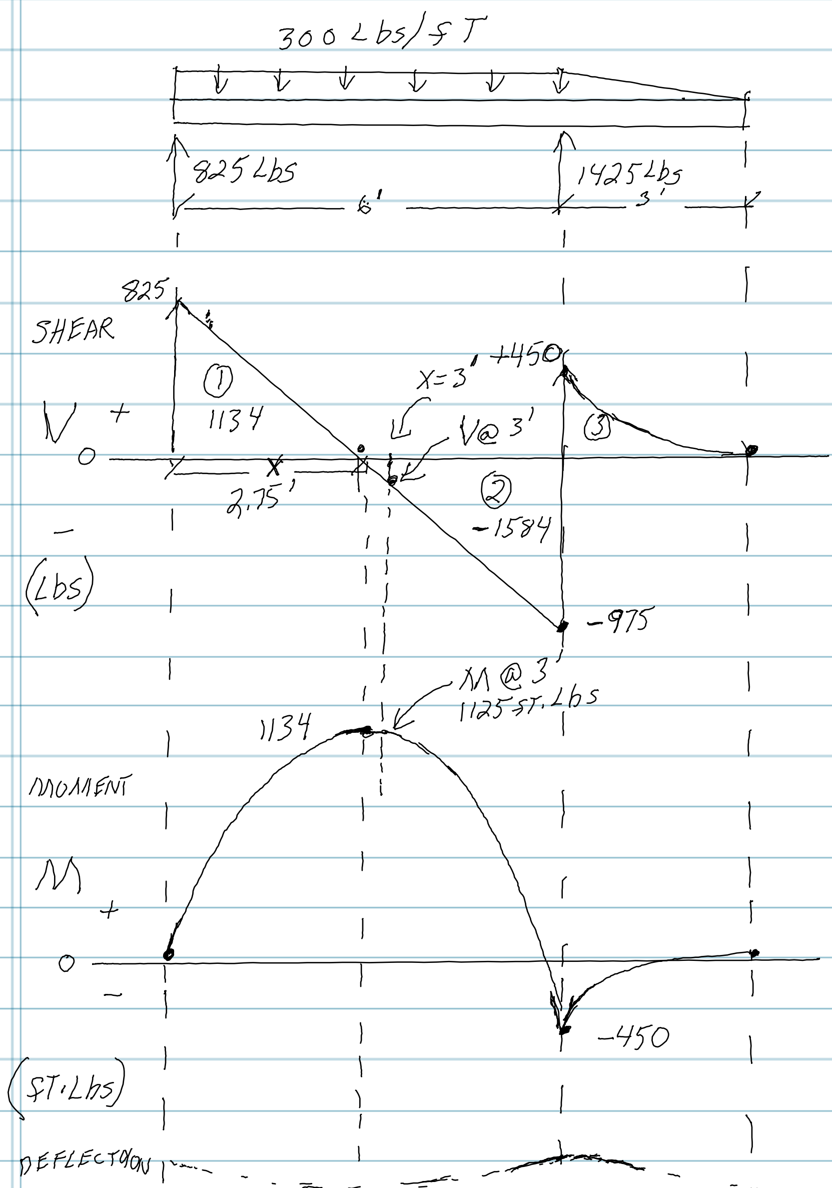
SHEAR + MOMENT DIAGRAM

PROBLEM F 1.5 (CONT.)

SHEAR & MOMENT DIAGRAMS



1) NOT IN TEXTBOOK

2) EXPLAIN - WHY "V" ↓

$V = -75$ Lbs

$y = mx + b$

$y = mx + 825$

$y = -300 \frac{Lbs}{FT} x + 825$ Lbs

$0 = -300x + 825$

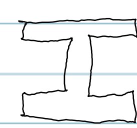
$x = \frac{825}{300} = 2.75'$

$A_1 = \frac{1}{2} bh = \frac{1}{2}(2.75')\ 825\ Lbs = 1134$ FT·Lbs

$A_2 = \frac{1}{2}(6' - 2.75)(-975)$

$A_2 = -1584$ FT·Lbs

TAKE AWAY - SHEAR & MOMENT - SKETCH OF ALL



SHEAR FORCE MAX. VALUE
975 Lbs - MAX LEFT SIDE

MOMENT - MAX 1134 FT·Lbs @ 2.75'
CROSS SECTION @ 3' 1125 FT·Lbs

NOT ON TEST OR QUIZ - MOST LOAD - POINT-UNIFORM

CH. 1.3 – 1.5 STRESS

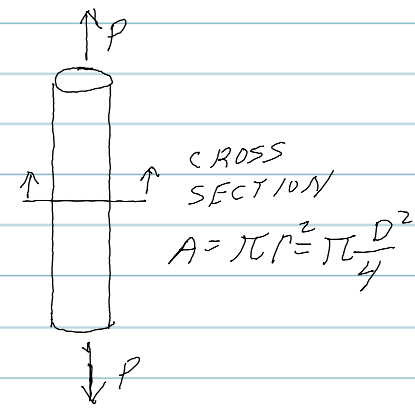
SECTION 1.3 DEFINITION

$$\sigma = \frac{P}{A}$$

P ← lbs, N

A ← in^2, m^2

σ ← PSI, Pa



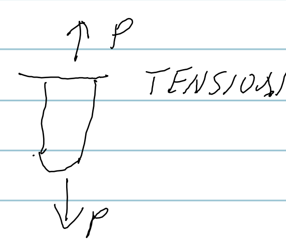
SECTION 1.4 NORMAL STRESS

"P" INTERNAL RESULTANT NORMAL FORCE (⊥ TO CROSS SECTION) – ACTING THROUGH CENTROID.

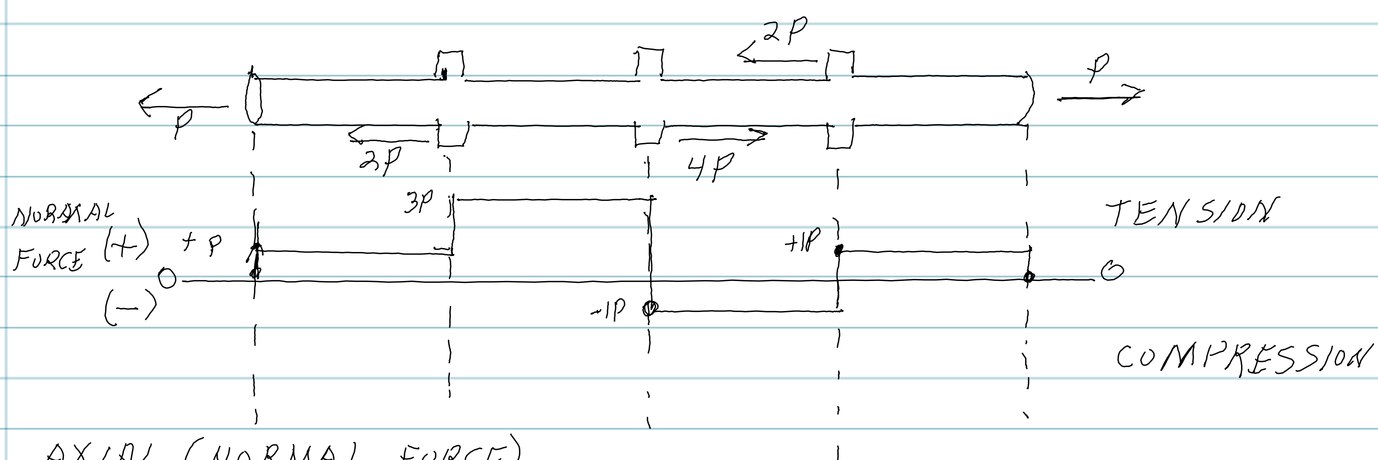
"A" AREA OF CROSS SECTION (USUALLY ⊥ TO LONG AXIS)

UNIAXIAL STRESS – FORCES ALONG 1 AXIS –
ONLY NORMAL FORCE (NO SHEAR FORCE)
TENSION OR COMPRESSION

$+\sigma_{NORMAL}$ – TENSION $-\sigma$



PURE TENSION or COMPRESSION
(ANY 2 FORCE MEMBER)



AXIAL (NORMAL FORCE) DIAGRAM

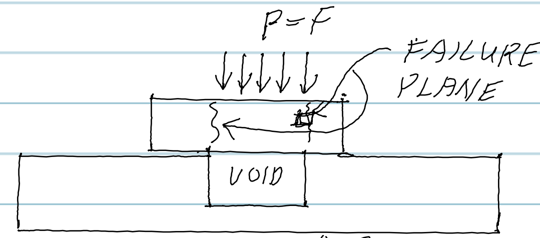
GET CONFUSED – DRAW FBD – CUT @ SECTION

CH. 1-3 TO 1-5 (CONT.) STRESS

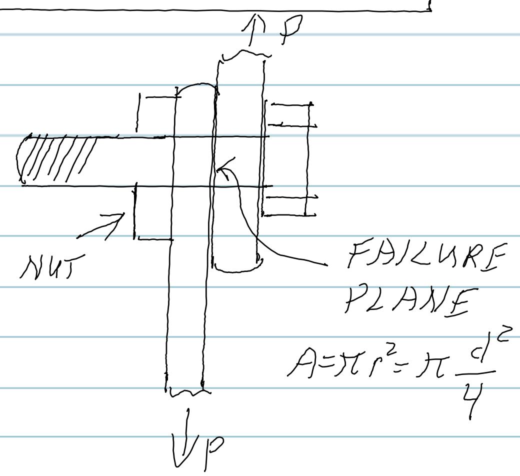
SECTION 1-5 AVERAGE SHEAR STRESS

$$\tau \equiv \frac{V}{A}$$

↖ FAILURE PLANE (TOTAL) (CROSS SECTION)

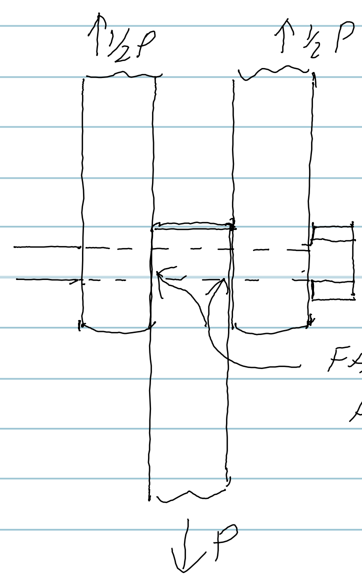


BOLT FAILURE – SINGLE SHEAR



$A = \pi r^2 = \pi \frac{d^2}{4}$

BOLT FAILURE DOUBLE SHEAR



FAILURE PLANES

$A_{FAILURE} = 2\,A_{BOLT}$

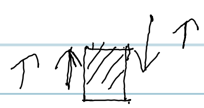
$$\tau = \frac{V}{A}$$

V ← Lbs, N

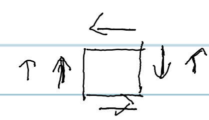
A ← in^2, m^2

τ ← PSI, Pa

SHEAR STRESS CAUSES OTHER SHEAR STRESSES (COMPLEMENTARY PROPERTY)



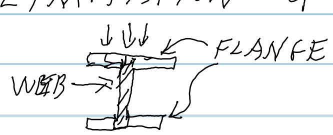
$\Sigma F_y = 0$ $\quad \Sigma M = 0$



$\Sigma F_x = 0$

PURE SHEAR (NO NORMAL FORCES)

LIMITATION OF AVERAGE SHEAR – ASSUMPTION IS NO BENDING.



CH. 1-3 TO 1.5 SHEAR (CONT.)

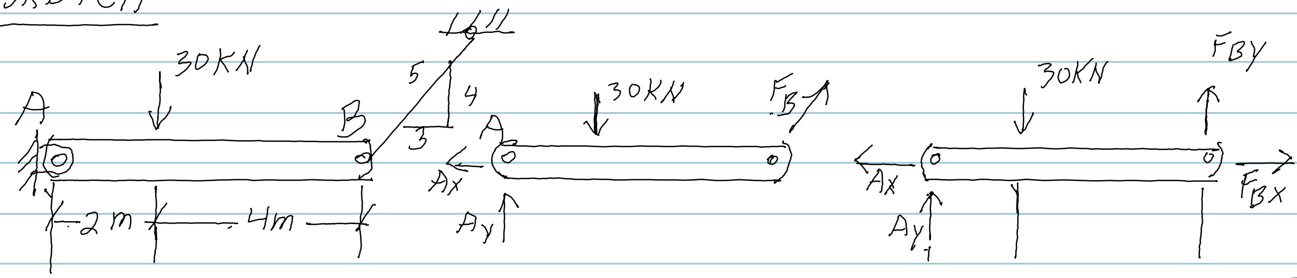
PROCEDURE TO FIND AVERAGE SHEAR STRESS

1) DRAW 1 or MORE FBD – INTERIOR – FBD CUT TO EXPOSE FORCES @ PLANE OF FAILURE
2) $\tau_{AVG} = \frac{V}{A}$
3) DRAW τ'S ON CUBE.

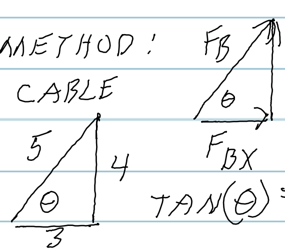
EXAMPLE PROBLEN 1.9 (SHOWN IN BOOK) USES PROPORTIONAL Δ'S

FIND: AVERAGE SHEAR STRESS IN PIN "A" (diameter = 20mm, double shear) + PIN "B" (diameter = 30mm, single shear).

SKETCH



OLD METHOD:



$SIN(\theta) = \frac{F_{BY}}{F_B}$

$F_{BY} = F_B\, SIN(\theta) =$

$TAN(\theta) = \frac{4}{3} \Rightarrow \theta$

$F_{BY} = F_B\, SIN(53.1^\circ) = .8 F_B$

NEW METHOD:

$\frac{F_B}{5} = \frac{F_{BY}}{4}$

$F_{BY} = \frac{4}{5} F_B = .8 F_B$

$F_{BX} = \frac{3}{5} F_B = .6 F_B$

GENERAL FORMULA:

$$F_y = \left[\frac{RISE}{\sqrt{RISE^2 + RUN^2}}\right] F$$

$$F_x = \left[\frac{RUN}{\sqrt{RISE^2 + RUN^2}}\right] F$$

CH. 1-3 TO 1-5 EXAMPLE 1.9 (CONT.)

SOLVE FBD: $\Sigma M_A = 0 \Rightarrow -(30\,kN)2m + (.8F_B)6m = 0$

$F_B = 12.5\ kN$

$\Sigma F_y = 0 \Rightarrow +A_y - 30 + .8F_B = 0$

$A_y = 20\ kN$

$\Sigma F_x = 0 \Rightarrow -A_x + .6F_B = 0 \Rightarrow A_x = 7.5\ kN$

SOLVE FOR SHEAR FORCE @ "V_A" "V_B"

FOR: $V_A = -7.5i + 20j\ (kN)$ $|V_A| = \sqrt{7.5^2 + 20^2} = 21.36\ kN$

$|V_B| = 20\ kN$

SOLVE FOR SHEAR FAILURE AREA'S

$A_A = (\text{DOUBLE SHEAR}) = 2 \cdot \pi r^2 = 2 \cdot \frac{\pi}{4} d_a^2 = 2 \cdot \frac{\pi}{4}\left(20mm\left(\frac{1m}{1000mm}\right)\right)^2$

$A_A = 6.28 \cdot 10^{-4} m^2$

$A_B = (\text{SINGLE SHEAR}) = \pi r_b^2 = \frac{\pi}{4} d_b^2 = \frac{\pi}{4}\left(30mm\left(\frac{1\ m}{1000\ mm}\right)\right)^2 = 7.09 \cdot 10^{-4} m^2$

SOLVE FOR SHEAR STRESS

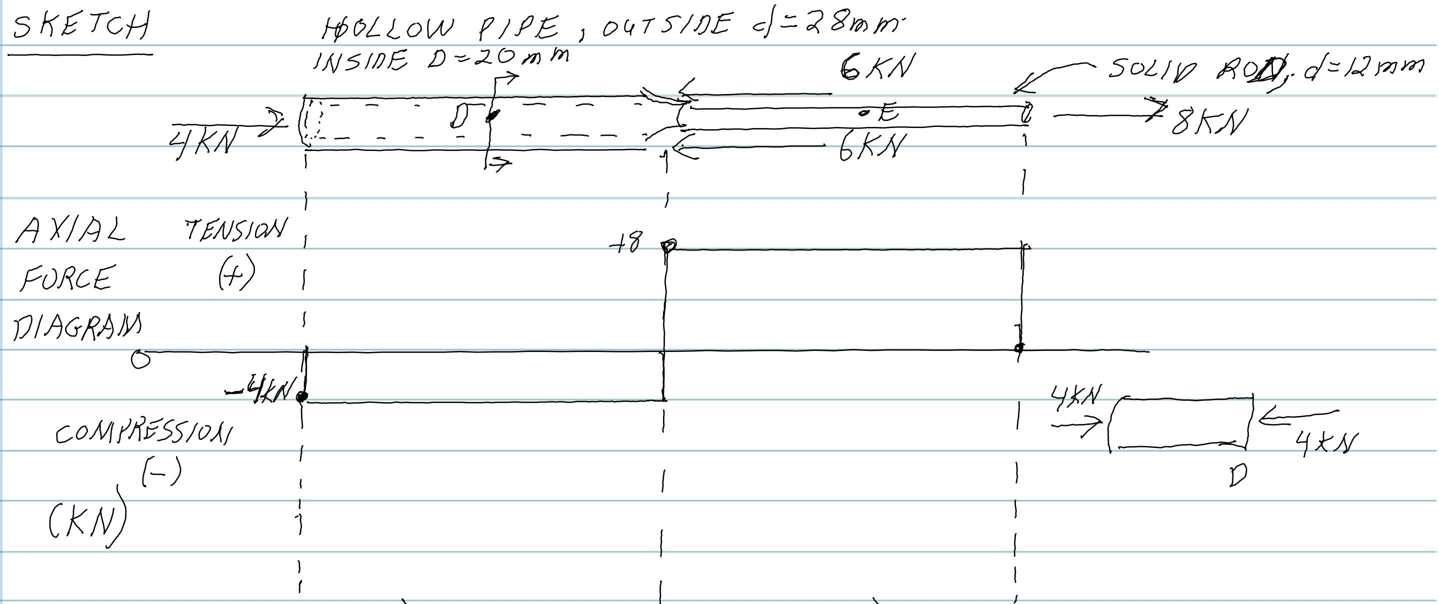
$\tau_A = \frac{V_a}{A_a} = \frac{21.36\,kN\left(\frac{1000\ N}{1\ kN}\right)}{6.28 \cdot 10^{-4} m^2} = 3.4 \cdot 10^7 Pa \left(\frac{1\ MPa}{10^6\ Pa}\right) = 34\ MPa$

$\tau_B = \frac{V_b}{A_b} = \frac{20kN\left(\frac{100\ N}{1\ kN}\right)}{7.09 \cdot 10^{-4} m^2} = 1.77 \cdot 10^7 Pa = 17.7\ MPa$

CH 1.3 - 1.5 PROB. 1-34

FIND: a) AVERAGE NORMAL STRESS @ "D" & "E"
b) DRAW STRESS ELEMENTS @ "D" & "E"

SKETCH



$N_D = 4\ KN\ (C)$, $N_E = 8\ KN\ (T)$

SOLVE FOR AREA @ "D" & "E"

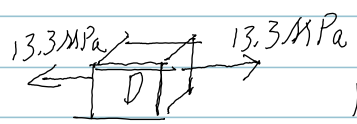
$$A_E = \pi r^2 = \frac{\pi}{4} d^2 = \frac{\pi}{4}(12mm)^2 = 113.1\ mm^2\left(\frac{1\ m}{1000mm}\right)^2 = 113.1 \cdot 10^{-6} m^2$$

$$A_D = \frac{\pi}{4} d^2 = \frac{\pi}{4}(28^2 - 20^2) = 301.6\ mm^2\left(\frac{1\ m}{1000\ mm}\right) = 301.6 \cdot 10^{-6}\ m^2$$

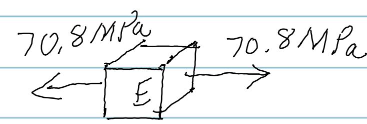
SOLVE FOR STRESS σ:

$$\sigma_D = \frac{N_D}{A_D} = \frac{4000\ N}{301.6 \cdot 10^{-6}\ m^2} = 1.33 \cdot 10^7 Pa\left(\frac{1\ MPa}{10^6\ Pa}\right) = \underline{13.3\ MPa}$$

$$\sigma_E = \frac{8000\ N}{113.1 \cdot 10^{-6} m^2} = 7.08 \cdot 10^7 Pa = \underline{70.8\ MPa}$$



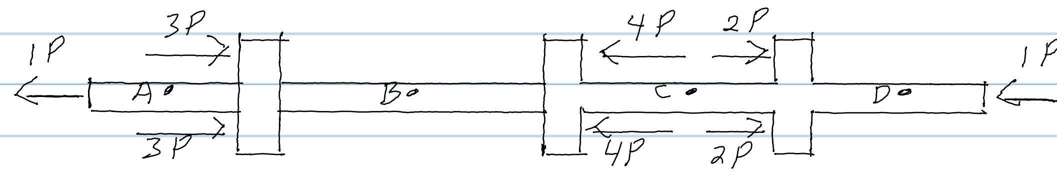
PURE NORMAL STRESS



DONE

CH 1.3 - 1.5 STRESS RANKING 2

RANK THE NORMAL STRESS @ CROSS SECTIONS A-D. THE PIPE HAS A CONSTANT CROSS SECTIONAL AREA WITH 8 DIFFERENT FORCES APPLIED AT 5 DIFFERENT LOCATIONS. RECALL THAN MATERIAL IN COMPRESSION IS POSITIVE, TENSION IS NEGATIVE.



GREATEST 1) ____
2) ____
3) ____
LEAST 4) ____
ALL THE SAME ____ ____

YOUR REANSONING!

CONFIDANT IN YOUR RANKING? (CIRCLE ONE)

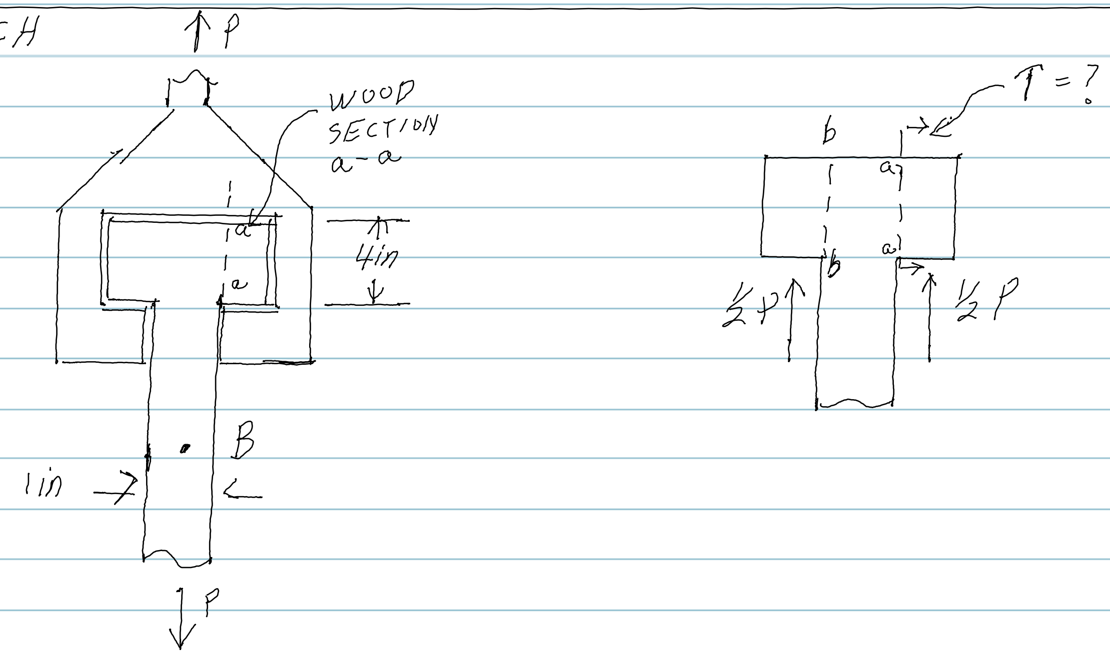
LOW GOOD POSITIVE

CH. 1-3 TO 1-5 PROB. 1-51

GIVEN: WOOD TEST SPECIMEN 2 in THICK (Z DIMENSION)
X-Y DIMENSIONS SHOWN ON SKETCH. NORMAL STRESS
(∥ TO LONG AXIS) IS 2 ksi @ SECTION "B".

FIND: a) FORCE "P" = ? b) V_{aa} = ?, τ = ?

SKETCH



SOLUTION:

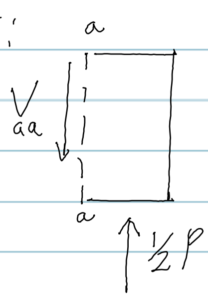
a) $\sigma_B = \frac{P_B}{A} \Rightarrow P_B = \sigma_a \cdot A = (2\ ksi)(1'' \times 2'') = \underline{\underline{4\ kips}}$

b) SHEAR FAILURE AREA $= 2 \cdot (4in \cdot 2in) = 16 in^2$

$V_{TOTAL} = 4\ KIPS$

$$\tau = \frac{V_{TOTAL}}{A_{TOTAL}} = \frac{4\ KIPS}{16 in^2} = .25\ ksi = \underline{\underline{250\ psi}}$$

ALT:



$V_{aa} = \frac{1}{2} P = \frac{1}{2}(4\ KIPS) = 2\ KIPS$

$A_{aa} = 4in \times 2in = 8 in^2$

$$\tau = \frac{V_{aa}}{A_{aa}} = \frac{2\ KIPS}{8 in^2} = \underline{\underline{250\ psi}}$$

CH 1.6 – 1.7 ASD + LRFD

ALLOWABLE STRESS DESIGN

$$F.S = \frac{F_{FAIL}}{F_{ALLOW}}$$

F_{FAIL} ← FORCE NECESSARY TO CAUSE FAILURE

F_{ALLOW} ← MAXIMUM SAFE FORCE

F.S ← FACTOR OF SAFETY

$$F.S. > 1$$

EXAMPLE: $F_{ALLOW} = \frac{F_{FINAL}}{F.S.} = \frac{4 \text{ kips}}{2} = \underline{\underline{2 \text{ KIP}}}$

RETAINING WALL – $F.S. \simeq 4$

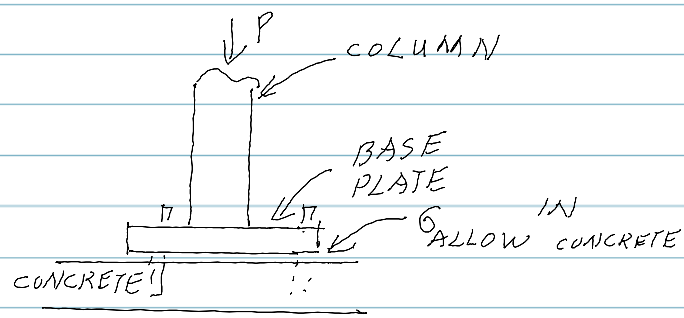
NUCLEAR PLANT – $F.S \sim 3$

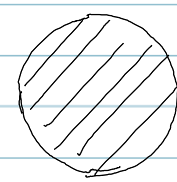
AIRCRAFT STRUCTURE – $F.S. \simeq 1.\sim$

$F.S. = \frac{\sigma_{FAIL}}{\sigma_{ALLOW}}$ OK WHEN $\sigma \simeq F$

$F.S. = \frac{\tau_{FAIL}}{\tau_{ALLOW}}$

APPLICATION: SIMPLE CONNECTION (NORMAL or SHEAR FORCE @ C.S.)





$$A_{BOLT} = \frac{V}{\tau_{ALLOW}}$$

$$A_{BP} = \frac{P}{\sigma_{ALLOW}}$$

CH 1,6 – 1,7

LOAD + RESISTANCE FACTOR DESIGN (LRFD)

MORE ACCURTE METHOD TO SIZE MATERIALS

SHOE SIZES – EUROPE – 1 # 46, 52,
U.S – 2 PARAMETERS

ASD VS LRFD — ASD – F.S.
LRFD = 2 PARAMETERS

HISTORY – TEST STRUCTURES

DEAD LOAD – FIXED – KNOW WEIGHTS – ~ CONSTANT
LIFE LOAD – PEOPLE, EQUP. – VARIABITY
SNOW LOAD –
WIND " –
EARTHQUAKE –

SPECIFICS – LRFD

STRENGTH SIDE : ϕP ← FORCE REQUIRED TO FAIL
$\phi < 1$

LOAD (RESISTANCE)

$$R = 1.2 D_L + 1.6 L_L + .5 S + \ldots$$

$$\phi P = R = 1.2 D_L + 1.6 L_L$$

PROCEDURE TO APPLY ASD or LRFD

1) FIND CRITICAL C.S (FAILURE WOULD OCCUR – OR DIRECTED)

2) ASD – F.S. ⇒ σ_{ALLOW} ⇒ $A_{MIN} = \dfrac{N}{\sigma_{ALLOW}} = \dfrac{V}{\tau_{ALLOW}}$

3) LRFD – FBD CUT C.S. → FACTOR LOAD → SOLVE FBD

CH 1.6 - 1.7 (CONT.)

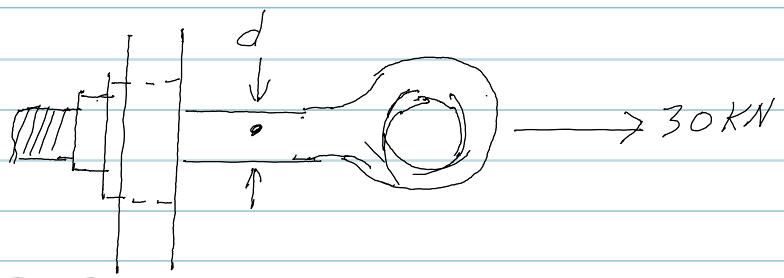
4) $P = \phi \sigma_{FAIL} A_{C.S.} \Rightarrow \phi \sigma_{FAIL} A = 1.2 D_L + 1.6 L_L$ ← SOLVE FOR

EX. PROB. F1-19 FIND: MINIMUM DIA (d=?) OF BOLT SHANK.

SKETCH

$\sigma_{FAIL} = 250$ MPa

F.S. = 1.5



SOLUTION: ASD METHOD

$$FS = \frac{\sigma_y}{\sigma_{ALLOW}} \Rightarrow \sigma_{ALLOW} = \frac{\sigma_y}{1.5} = \frac{250 \cdot \text{MPa}}{1.5} = 166.7 \text{ MPa}$$

$$\sigma_{ALLOW} = \frac{N}{A_{CS}} \Rightarrow A = \frac{N}{\sigma_{ALLOW}} = \frac{30 \cdot 10^3 N}{166.7 \cdot 10^6 Pa} = 1.80 \cdot 10^{-4} m^2$$

$$A = \frac{\pi}{4} d^2 \Rightarrow d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(1.8 \cdot 10^{-4})}{\pi}} = .0151 m = 15.1 mm$$

OTHER FAILURE MODES

1) EYE BREAK - SHEAR, NORMAL FORCE, or COMBO.
2) THREADS/NUT -
3) WASHER FAIL - BENDING - PULL OUT THRU WALL
4) WALL MATERIAL FAIL IN SHEAR

NOTE: LRFD - BOOK DOES NOT CONTAIN PROB.

NO QUIZ OR TEST ON LRFD.