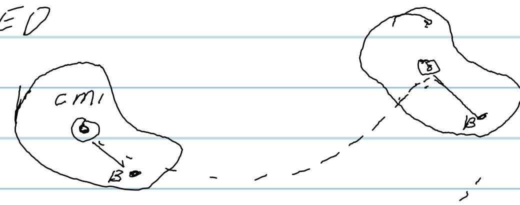
CH. 17 PLANAR KINEMATICS OF RIGID BODIES

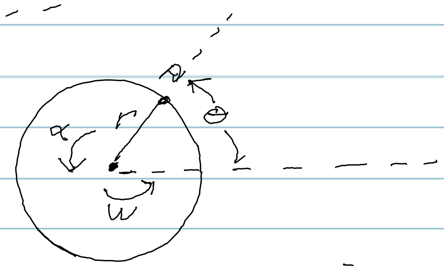
SECTION 17-1 + 2 TYPES OF MOTION - ROTATIONS

RIGID BODY: CM IS FIXED

TRANSLATION:



ROTATION:



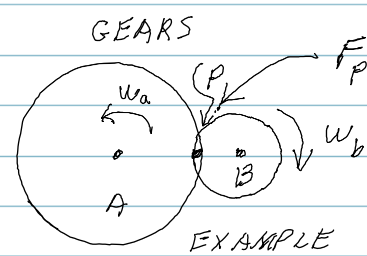
- ALL POINT ON AN OBJECT ROTATE W/ SAME - θ, ω, α

- ARC LENGTH

$$s = r\theta$$
$$v = r\omega$$
$$a_T = r\alpha$$

$$a_T = r\alpha \qquad a_n = \frac{v^2}{r} = r\omega^2$$

GEARS



- GEAR RATIO'S

$$r_a\theta_a = \underline{s_a = s_b} = r_b\theta_b$$

$$r_a\theta_a = r_b\theta_b \Longrightarrow \frac{r_a}{r_b} = \frac{\theta_b}{\theta_a}$$

$$r_a\omega_a \overset{v_p}{=} r_b\omega_b \qquad \frac{r_a}{r_b} = \frac{\omega_b}{\omega_a}$$

$$r_a\alpha_a \overset{a_T}{=} r_b\alpha_b \qquad \frac{r_a}{r_b} = \frac{\alpha_b}{\alpha_a}$$

EXAMPLE $3B_r = 1A_r$

$$\frac{3}{1} = \frac{\theta_b}{\theta_a}$$ ← rad, or θ, rev

INVERSE RATIO

$$\frac{r_a}{r_b} = \frac{\text{"}d_a\text{"}}{\text{"}d_b\text{"}} = \frac{C_a}{C_b} = \frac{\#\,TEETH_a}{\#\,TEETH_b} \equiv \underbrace{\frac{\theta_b}{\theta_a} = \frac{\omega_b}{\omega_a} = \frac{\alpha_b}{\alpha_a}}_{} = \frac{T_a}{T_b}$$

- TORQUE ON SHAFTS

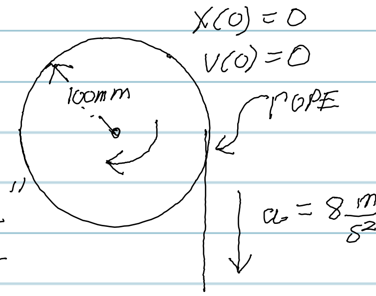
TORQUE, MOMENT

$$T = M = r \times F = r_\perp F$$

$$\frac{T_a}{r_a} = F_p = \frac{T_b}{r_b} \Longrightarrow \frac{r_a}{r_b} = \frac{T_a}{T_b}$$ } DIRECTLY PROPORTIONAL

- $F = \frac{T}{r}, \quad a_T = r\alpha, \quad a_n = \frac{v^2}{r} = r\omega^2$

SECTION 17-2 HOMEWORK EXAMPLE



17-3 GIVEN ⇒ $r = 100mm = 0.1m$, $a_T = 8\,\frac{m}{s^2}$

a) FIND: $W(1m) = ?$

SOLUTION:

$V = rW \implies V(1m) = ?$ "CONSTANT ACCEL" TYPE IN CONSTANT - (Pg 103)

$V^2 = V_i^2 + 2a(X - X_0)$

$V_i = 0$, $V = ?$, $a = 8\,m/s^2$, $T =$, $x = 0$, $X_0 = 1m$

$V^2 = 0^2 + 2(8\,\frac{m}{s^2})(1m - 0) = 16\,\frac{m^2}{s^2}$

$V(1m) = 4\,\frac{m}{s}$

$W(1m) = \frac{V(1m)}{r} = \frac{4\,m/s}{0.1m} = \underline{\underline{40\,\frac{rad}{sec}}}\left(\frac{1.047\,RPM}{10\,\frac{rad}{sec}}\right) = 4.2\,RPM\text{'s}$

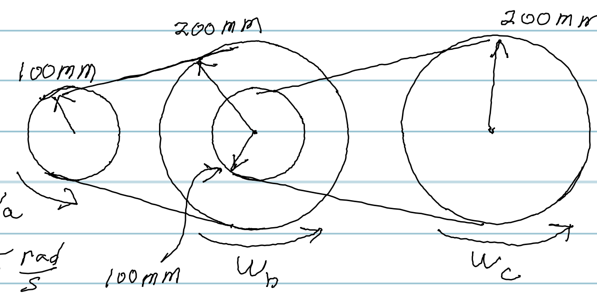
SHOULD HAVE BEEN a)

b) FIND: $a_T(1m) = ?$ $\quad \underline{\underline{a_T = 8\,\frac{m}{s^2}}}$

FIND: $a_N(1m) = ?$ $\quad a_n = \frac{V^2}{r} = \frac{(4\,m/s)^2}{0.1m} = \underline{\underline{160.\,\frac{m}{s^2}}}$

17-5 GIVEN: $r_a = .1m$, $r_{bOUT} = .1m$, $r_{bIN} = .2m$, $r_c = .2m$, $W_a = 4 + .2T$ $(\frac{rad}{s})$

FIND: $W_b(5s) = ?$ $\quad W_c(5s) = ?$



SOLUTION: $W_a = 4 + .2(5s) = 5\,\frac{rad}{s}$

a) $\frac{r_a}{r_{bIN}} = \frac{W_b}{W_a} \Rightarrow \frac{1}{2} = \frac{W_b}{5\,rad} \Rightarrow \underline{\underline{W_b = 2.5\,\frac{rad}{s}}}$

$\frac{r_{bOU}}{r_c} = \frac{W_c}{W_b} \Rightarrow \frac{.1m}{.2m} = \frac{W_c}{2.5\,\frac{rad}{s}} \Rightarrow \underline{\underline{W_c = 1.25\,\frac{rad}{s}}}$

b) FIND: $\theta_c(5s) = ?$ $\quad$ ① $\frac{\theta_b}{\theta_a} = \frac{W_b}{W_a} = \frac{\alpha_b}{\alpha_a} \Rightarrow \frac{W_c}{W_a} = \frac{\theta_c}{\theta_a} \Rightarrow \frac{1.25}{5} = \frac{\theta_c}{\theta_a} \Leftarrow ?$

② $\frac{d\theta_a}{dt} = W(T) \Rightarrow \int_0^{\theta} d\theta_c = \int_0^{T=5sec} W(T)\,dT = \int_0^{T=5} (4 + .2T)\,dT = 4T + .1T^2\Big|^{T=5}$

$\theta_a = 4T + .1T^2\Big|_0^{5sec} = 4(5) + .1(5)^2 = 22.5\,rad$

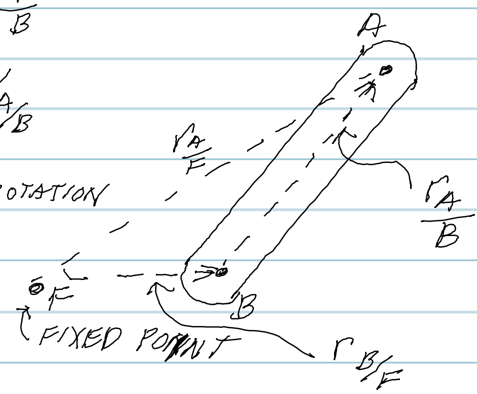
① $\frac{1.25\,\frac{r}{s}}{5\,rad} = \frac{\theta_c}{22.5\,rad} \Rightarrow \underline{\underline{\theta_c = 5.6\,rad}}$

SECTION 17-3 GENERAL MOTIONS: TRANSLATION + ROTATION

RELATIVE VELOCITIES

$$r_{A/F} = r_{B/F} + r_{A/B}$$

$$V_{A/F} = \underbrace{V_{B/F}}_{\text{TRANSLATION}} + \underbrace{V_{A/B}}_{\text{ROTATION}}$$

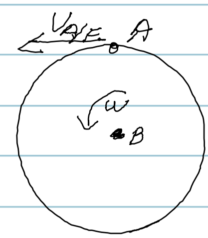


• ROTATION ONLY

• $V_{A/F} = V_{B/F} + V_{A/B}$ "B" IS FIXED $\Rightarrow V_{B/F} = 0$

$V_{A/F} = 0 + V_{A/B}$ $V = r\omega$

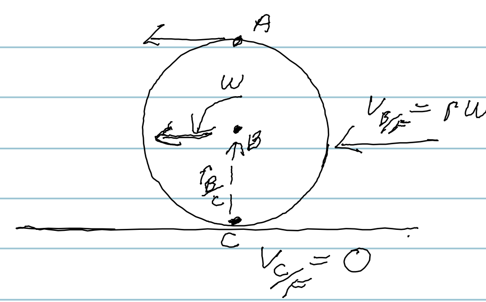
$V_{A/F} = r\omega\, i$



• ROLLING (NO SLIP)

$V_{B/F} = V_{C/F} + V_{B/C} = 0 + (-r_{B/C}\,\omega i)$

$V_{B/F} = -r\omega i$



• VELOCITY AT "A" (TOP OF TIRE)

$$V_{A/F} = V_{B/F} + V_{A/B} = -r\omega i + (-r\omega i) = \underline{\underline{-2r\omega i = -2Vi}}$$

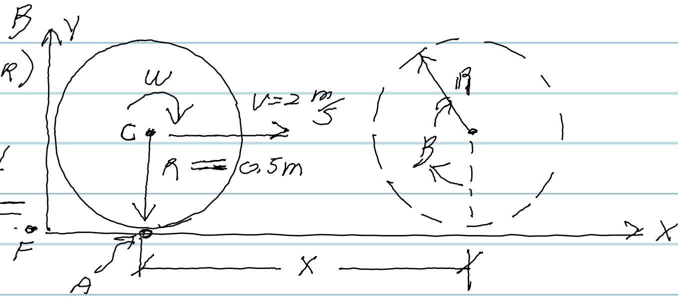
• 3-D ROTATION

$V_{A/B} = \omega \times r_{A/B}$

SECTION 17-3 HOMEWORK EXAMPLES

17-17 ROLLING DISK GIVEN: $X = R\beta$

FIND: $|\omega| = ?$, $\omega = ?$ (VECTOR)



SOLUTION:

a) $V = r\omega \Rightarrow \omega = \frac{V}{r} = \frac{2\ m/s}{0.5\ m} = 4\ \frac{rad}{s}$

b) $\omega = ?$ RIGHT HAND RULE

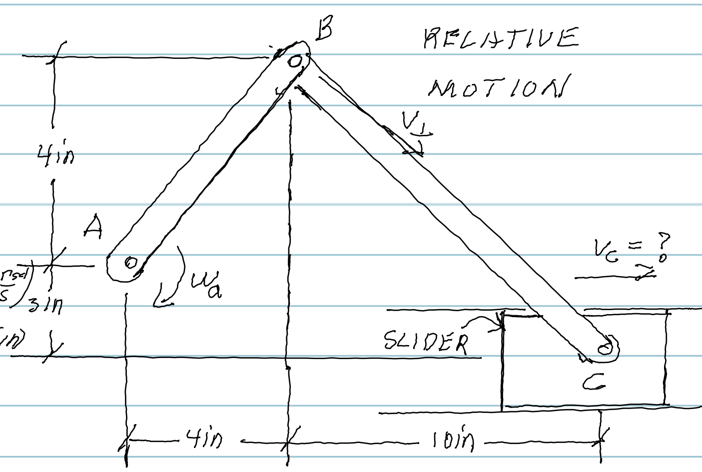
$\omega = (-4K)\ \frac{rad}{s}$

ALTERNATE SOLUTION: BY RELATIVE VELOCITIES

$$V_{C/F} = V_{A/F} + V_{C/A} = 0 + 2\ m/s = V_{A/F} + r_{C/A}\,\omega \Rightarrow 2\tfrac{m}{s} = r\omega$$

17-31 GIVEN: $\omega_a = 6\ \frac{rad}{s}$

FIND: $V_c = V_{cx}\,i + 0_y\,j = ?$



SOLUTION: $V_A = 0$

$V_{B/F} = \underbrace{V_{A/F}}_{\text{TRANSLATION}} + \underbrace{V_{B/A}}_{\text{ROTATIONAL}} = 0$

$V_B = \omega_a \times r_{B/A} \Rightarrow \omega_a = 0i + 0j - 6K\ (\frac{rad}{s})$

$r_{B/A} = 4i + 4j + 0K$ (in)

$$V_B = \omega_a \times r_{B/A} = \begin{array}{c} \\ \omega_a \\ r_{B/A} \end{array}\begin{vmatrix} i & j & K \\ 0 & 0 & -6 \\ 4 & 4 & 0 \end{vmatrix}\begin{matrix} i & j \\ 0 & 0 \\ 4 & 4 \end{matrix}$$

$$V_B = -24j - [-24i] = 24i - 24j$$

$V_C = V_b + V_{C/b} = V_b + \omega_{bc} \times r_{C/b} \Rightarrow r_{C/b} = +10i - 7j$ (in)

$\omega_{bc} = 0i + 0j + \omega_z K$

$$V_{C/b} = \omega_{bc} \times r_{C/b} = \begin{vmatrix} i & j & K \\ 0 & 0 & \omega_z \\ +10 & -7 & 0 \end{vmatrix}\begin{matrix} i & j \\ 0 & 0 \\ 10 & -7 \end{matrix} = 10\omega_z j - [-7\omega_z i]$$

$$V_{C/b} = 7\omega_z i + 10\omega_z j \text{ (in)}$$

SECTION 17-3 HOMEWORK EXAMPLE 17-31 CONT.

• $V_C = V_B + V_{C/B}$

$$
\begin{aligned}
V_B &= \overset{①}{24i} \quad \overset{②}{-24j} + 0K \\
+\ V_{C/B} &= 7\omega_z i \quad +10\omega_z j + 0K \\
\hline
V_C &= V_{cx} i \quad + 0j \quad + 0K
\end{aligned}
$$

② $-24 + 10\omega_z = 0$

$10\,\omega_z = 24$

$\omega_z = 2.4\ \frac{rad}{s}$ (+ ANGULAR VELOCITY - CCW ROTATION)

① $24 + 7\omega_z = V_{cx}$

$V_{cx} = 24 + 7\left(2.4\frac{rad}{s}\right) = 40.8\ \frac{in}{s}$ $\qquad \vec{V}_C = 40.8i + 0j + 0K \left(\frac{in}{s}\right)$

INSTANTANEOUS CENTERS APPROACH TO PREVIOUS (ABOVE) PROBLEM

17-31 - I.C.

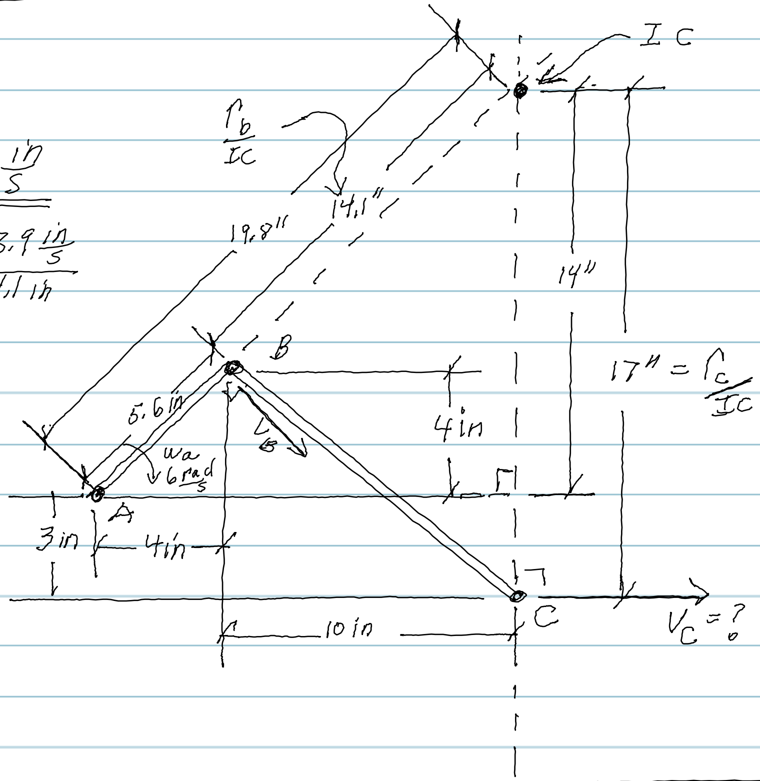
$|V_b| = r_{AB}\,\omega = (5.65\,in)\,6\frac{rad}{s} = 33.9\ \frac{in}{s}$

• $V_b = r_{b/IC}\,\omega_{BC} \Rightarrow \omega_{BC} = \frac{V_b}{r_{b/IC}} = \frac{33.9\frac{in}{s}}{14.1\,in}$

$\omega_{bc} = 2.4\ \frac{rad}{s}$

$V_C = r_{C/IC}\,\omega_{bc} = (17'')\left(2.4\frac{rad}{s}\right)$

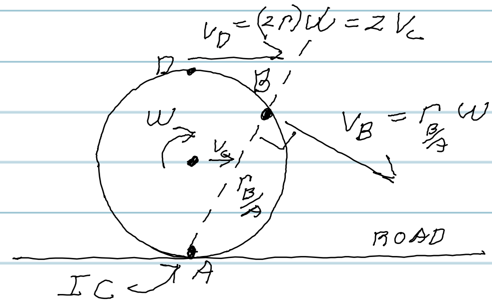
$V_C = 40.9\ \frac{in}{s}$



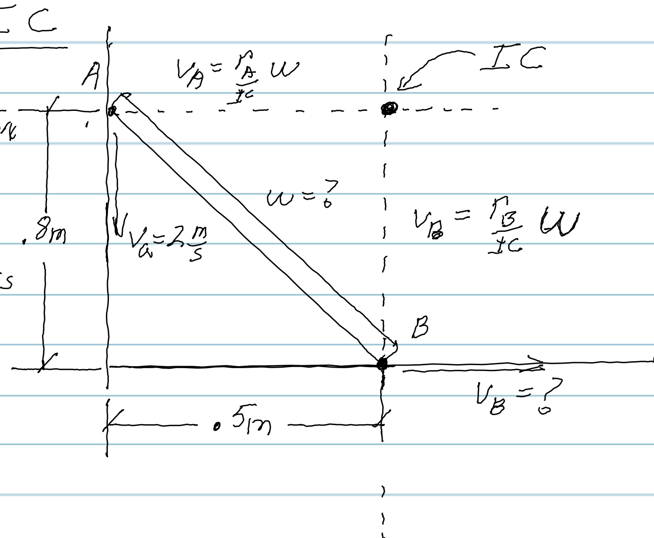
SECTION 17-4 INSTANTANEOUS CENTERS NOTES

IC – SNAPSHOT

- WHEN YOU KNOW IC $\Rightarrow V_P = \left(r_{P/IC}\right)(\omega)$



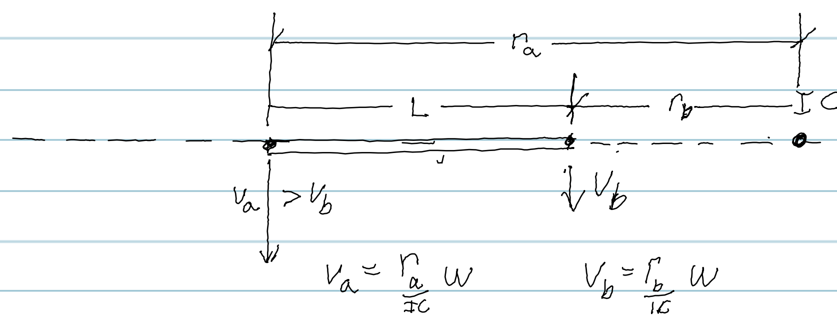
- WHEN YOU DON'T KNOW – IC
  - – ONE MEMBER – FIND DIRECTION OF 2 VELOCITY VECTORS
  - – CONSTRUCT $\perp$'s TO VELOCITIES
  - – INTERSECTION IS IC
  - – USE RELATION $V = r_{IC} \cdot \omega$



EXAMPLE: $\omega_{AB} = ?$ $\quad V_A = r_a \omega \Rightarrow \omega = \frac{V_a}{r} = \frac{2\,m/s}{.5\,m} = 4\ rad/s$

$V_b = ? \Rightarrow V_b = r_{b/IC}\,\omega = (.8m)\,4\frac{rad}{s} = 3.2\frac{m}{s}$

EXAMPLE 2:



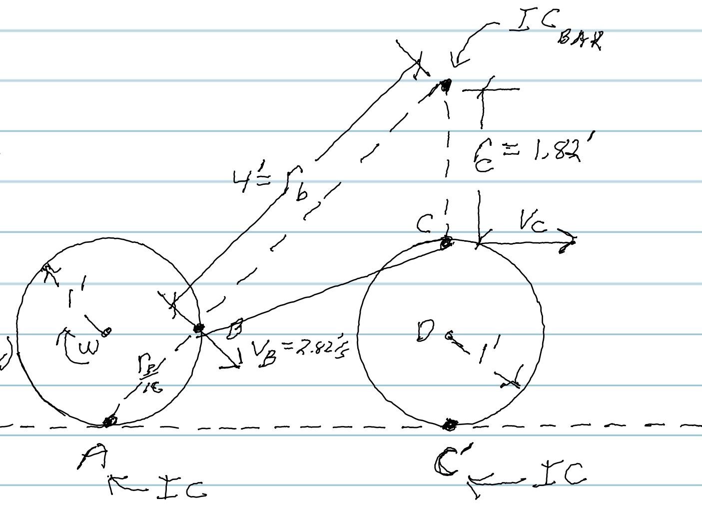
SECTION 17-4 INSTANTANEOUS CENTERS HOMEWORK EXAMPLE

17-77 I.C. GIVEN: $r = 1\ ft$, $\omega_A = 2\ \frac{rad}{s}$ CW

FIND: $\omega_{BAR} = ?$, $\omega_C = ?$

SOLUTION: • FIRST FIND IC's

• FIND 2 VEL. VECTORS



① $V_B = r_{B/IC}\,\omega = (1.41\ ft)\,2\ \frac{rad}{s} = 2.82\ \frac{ft}{s}$

② $\omega_{bar} = \frac{V_b}{r_{bar/IC}} = \frac{2.82\ \frac{ft}{s}}{4'} = .705\ \frac{rad}{s}$ (CCW)

③ $V_C = r_{bar_C/IC}\,\omega_{bar} = (1.82')\,(.705\ \frac{rad}{s})$

$V_C = 1.28\ \frac{ft}{s}$

④ $\omega_C = \frac{V_C}{r_{DISK/IC}} = \frac{1.28\ \frac{ft}{s}}{2'} = 0.64\ \left(\frac{rad}{s}\right)$ CW