CH. 19 ENERGY + MOMENTUM IN RIGID BODY DYNAMICS

SECTION 19-1 WORK + ENERGY

WORK = TRANSLATION:

$$U_{12} = W = F_{\parallel} d = F \cdot r \;\text{(DOT PRODUCT)} = |F||r|\cos(\theta) \quad \{\text{ASSUMPTION } \theta = \text{CONSTANT}\}$$

U_{12} JOULE

$$U_{12} = W = \int F \cdot dr$$

ROTATION: $U_{12} = W_r = M\theta = M\Delta\theta_{1-2} = \int M\,d\theta$

W_r JOULES, FT·LBS

POWER = TRANSLATION:

$$P = \frac{\Delta W}{T} = \frac{\Delta U_{12}}{T} = \frac{F \cdot r}{T} = F \cdot V = |F||V|\cos\theta \quad \{\theta = \text{CONSTANT}\}$$

P WATTS

ROTATION:

$$P_r = M\omega = \int M\,d\omega \quad \{2\text{ D CASE}\}$$

M MOMENT APPLIED, ω ANGULAR VELOCITY

AVERAGE POWER

T_2, T_1 KENETIC ENG OF A SYSTEM (NOT CONSIDERING CONSERVATIVE FORCES)

$$P_{avg} = \frac{T_2 - T_1}{\Delta T} = \frac{TE_1 - TE_2}{\Delta T}$$

CONSERVATION OF ENERGY {INCLUDES - PRINCIPLE OF WORK + ENERGY}

$$TE_1 + W = TE_2$$

$$KE_1 + RE_1 + PE_1 + W = KE_2 + RE_2 + PE_2$$

$$KE = \tfrac{1}{2} m V_{cm}^2 \qquad RE_1 = \tfrac{1}{2} I_{cm}\,\omega^2 \qquad PE = mgh = \tfrac{1}{2} k x^2$$

KINEMATIC RELATIONSHIPS - DEFINITIONS $V_f^2 = V_i^2 + 2as$ $\quad s = r\theta$

$a^2 + b^2 = c^2$ (RIGHT TRIANGLE), LAW OF SINES, ROLLING

$V = r\omega$

$a_t = r\alpha$

SECTION 19-1 NOTES (CONT.)

- NOTE EXAMPLE 19.2 – WORKED OUT Pg. 425.

CRITICAL THINKING NOTE.

$TE_1 + W = TE_2$

$\cancel{PE_1} + 0_1 + M\theta = KE_{CM} + RE_{FW} + RE_{RW} + \cancel{PE_2}$

$M\theta = 2\left(\frac{1}{2} I_{CM} \omega^2\right) + \frac{1}{2} m V_{cm}^2$

$V = r\omega$

$s = r\theta$

19-6 HOMEWORK EXAMPLE: $I = 540\ kg \cdot m^2$, $M = 6500 - 20\theta$ (N·m)

$\theta = 10\ rev\left(\frac{2\pi\ rad}{1\ rev}\right) = 62.8\ rad$

FIND: $\omega \stackrel{?}{=}$ RPM @ $\theta = 62.8\ rad$

SOLUTION: CONSERVATION OF ENERGY EQUATION

$TE_1 + W = TE_2$

$PE_1 + 0 + W_r = PE_2 + \cancel{KE_T} + RE_2 \Rightarrow (1)\ W_r = \frac{1}{2} I \omega^2$

$W_r = M\theta = \int_0^{\theta = 10 rev = 62.8 rad} M\, d\theta = \int_0^{\theta} (6500 - 20\theta)\, d\theta = 6500\theta - 10\theta^2 \Big|_0^{\theta = 62.8}$

$W_r = 3.69 \cdot 10^5$ N·m (J)

NOW USING EQUATION #(1) $W_r = \frac{1}{2} I \omega^2$

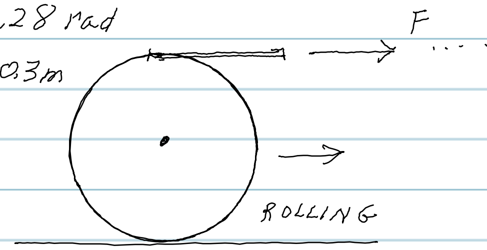
$\omega = \sqrt{\frac{2 W_r}{I}} = \sqrt{\frac{2(3.69 \cdot 10^5\ J)}{540\ kg \cdot m^2}} = 37\ \frac{rad}{sec}\left(\frac{1\ RPM}{1.047\ \frac{rad}{s}}\right) = 353\ RPM$

SECTION 19-1 HOMEWORK EXAMPLES (CONT.)

19-22 GIVEN: SOLID DISK, $F = 500N$, $\theta = 1\,rev = 6.28\,rad$

$I_{cm} = \frac{1}{2} mr^2$, $m = 100kg$, $r = 300mm = 0.3m$

FIND: $\omega = ?$ @ $\theta = 1$ rev.



SOLUTION: CONSERVATION OF ENERGY

$$0 + W = TE_F = KE_{T2} + RE_2$$

① $W = \frac{1}{2} m V_2^2 + \frac{1}{2} I \omega_2^2$

$W = F \cdot d = F(2 \cdot CIRM) = F \cdot 2 \cdot 2\pi r = (4\pi F r) = 4\pi \cdot 500N \cdot 0.3m$

$W = 1885\ J$

① $1885\,J = \frac{1}{2} mV^2 + \frac{1}{2} I\omega^2$ KINEMATIC EQ.: $V = r\omega$

$1885\ J = \frac{1}{2} m(r\omega)^2 + \frac{1}{2}\left[\frac{1}{2} mr^2\right]\omega^2 = \frac{1}{2} mr^2\omega^2 + \frac{1}{4} mr^2\omega^2 = \frac{3}{4} mr^2\omega^2$

$$\omega^2 = \frac{1885\ J}{\frac{3}{4} mr^2} = \frac{1885\ J}{.75(100kg)(0.3m)^2} = 279.3 \frac{rad^2}{s^2}$$

$$\omega = 16.7 \frac{rad}{s} \quad CW$$

SECTION 19-2 IMPULSE + MOMENTUM NOTES

LINEAR IMPULSE + MOMENTUM

$F\Delta T = m\Delta V = \Delta mV$

$P = mV$

$\int F\,dT = m\Delta V$

ANGULAR IMPULSE + MOMENTUM

$M_{cm}\Delta T = I_{cm}\Delta W = \Delta I_{cm}W$

$H_{cm} = I_{cm}W$

$\int M\,dT = \Delta I_{cm}W$

ISOLATED SYSTEM

MOMENTUM IS CONSERVED

$\Sigma mV_1 = \Sigma mV_2$

ANGULAR IS CONSERVED ABOUT ANY FIXED POINT $- H_0 \Rightarrow \Sigma H_{01} = \Sigma H_{02}$

IF COMMON CM (CENTER OF MASS)

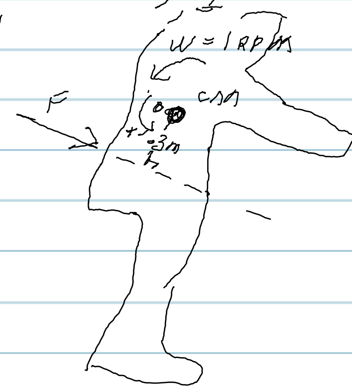
$H_0 = I_{cm}W \Rightarrow I_1 W_1 = I_2 W_2$

SECTION 19-2 HOMEWORK EXAMPLES

19-51 GIVEN: $m = 122\,kg$, $I_{cm} = 45.\ kg \cdot m^2$, $\Delta T = 0.2\,s$, $r_1 = 300\,mm = 0.3\,m$

$$\omega = 1.\ RPM\left(\frac{1.047\ \frac{rad}{s}}{10\ RPM}\right) = .1047\ \frac{rad}{s}$$

FIND: $F_{AV} = ?$ $V_{cm2} = ?$



SOLUTION: $\Sigma M_{cm} \Delta T = \Delta I_{cm} \omega = I\omega_2 - \cancel{I\omega_1}^{0}$

$$+ (F)(.3m)\,\Delta T = I\omega_2$$

$$F = \frac{I\omega_2}{.3\,\Delta T} = \frac{(45\ kg \cdot m^2)(0.1047\ \frac{rad}{s})}{(0.3m)(0.2\,s)} = \underline{\underline{78.5\ N}}$$

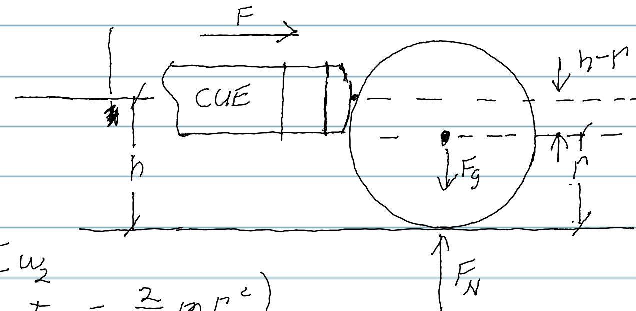
b) FIND: $V_{cm} = ?$

SOLUTION: LINEAR IMPULSE $F\Delta T = m\Delta V = mV_2 - \cancel{mV_1}^{0}$

$$V_2 = \frac{F\Delta T}{m} = \frac{(78.5N)(0.2s)}{122\ kg} = \underline{\underline{0.13\ \frac{m}{s}}}$$

19-57 GIVEN: $F_F = 0$ (NO SLIPPING)

FIND: $h = ?$ IN TERMS OF "r"



SOLUTION: THINGS THAT ARE TRUE

LINEAR: $F\Delta T = m\Delta V = mV_2 - \cancel{mV_1}^{0} = mV_2$

① $F\Delta T = mV_2$

ANGULAR: $M\Delta T = I\Delta\omega = I\omega_2 - \cancel{I\omega_1}^{0} = I\omega_2$

$(F)(h-r)\,\Delta T = I_{cm}\omega_2$ (NOTE: P_g 607 $I_{cm} = \frac{2}{5} m r^2$)

② $(F)(h-r)\,\Delta T = \frac{2}{5} m r^2 \omega_2 \Rightarrow F\Delta T = \dfrac{\frac{2}{5} m r^2 \omega_2}{(h-r)}$

COMBINE ① + ② USE $F\Delta T$

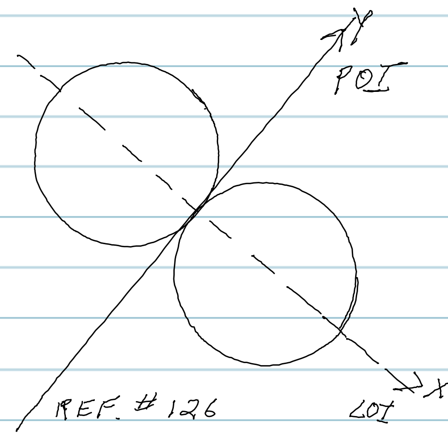
$$\cancel{m} V_2 = \frac{\frac{2}{5}\cancel{m} r^2 \omega_2}{(h-r)} \qquad \text{(KINEMATIC EQUATION } V = r\omega\text{)}$$

$$\cancel{r\omega_2} = \frac{\frac{2}{5} r \cancel{r \omega_2}}{h-r} \Rightarrow 1 = \frac{\frac{2}{5} r}{(h-r)} \Rightarrow (h-r) = \frac{2}{5} r$$

$$\underline{\underline{h = r + \frac{2}{5} r = \frac{7}{5} r = 1.4\,r}} \ : \ 40\% \text{ ABOVE "r"}$$

SECTION 19-3 IMPACTS

COEFICIENT OF RESTITUTION



$$e = \frac{\text{SPERATION VELOCITY}}{\text{APPROACH VELOCITY}} = \frac{V_{BP}' - V_{AP}'}{V_{AP} - V_{BP}}$$

ALONG LOI

ELASTIC PROCESS

$e = 1$ FULLY ELASTIC $\rightarrow$ $KE_b = KE_a$

$e = 0$ FULLY PLASTIC $\rightarrow$ OBJECT STICK TOGETHER

RESTRICT OUR DISCUSSION TO IMPACTS W/ A FIXED POINT.

SPECIAL CASES

1 – ISOLATED SYSTEM $\Sigma H_{O_B} = \Sigma H_{O_A}$ FIXED

2 – FIXED POINT ON ONE OBJECT – $\Sigma H_{O_B} = \Sigma H_{O_A}$

SECTION 19-3 HOMEWORK EXAMPLES

19-65 GIVEN: $W = F_g = 10\,lbs$, $e = 0.6$, $\theta_R = 10^\circ$, $\Delta T = 0.01s$

FIND: $F_{AV} = ?$

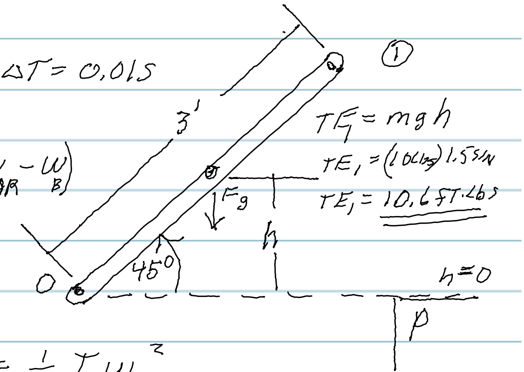
SOLUTION - PLAN: $\Sigma M \Delta T = \Delta H = \Delta I_o \omega = I(\omega_{AR} - \omega_B)$

$\omega_{AR} = ?$, $\omega_B = ?$

① $TE_1 = mgh$

$TE_1 = (10lbs)1.5\sin$

$TE_1 = 10.6\,ft\cdot lbs$



STEP 1 → $\omega_{before} = ?$

$TE_1 = TE_2$

$10.6\,ft\cdot lbs = \frac{1}{2} I \omega_b^2 = .466\,\omega_b^2$

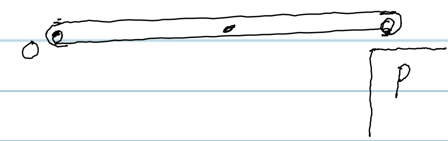
$\omega_b = 4.77\,\frac{rad}{s}$ CW

② $TE_2 = \frac{1}{2} I_o \omega_b^2$

$I_o = \frac{1}{3} mL^2$

$I_o = \frac{1}{3}\left(\frac{10lb}{32.2}\right)(3')^2$

$I_o = 0.932\,sl\cdot ft^2$



STEP 2 → $\omega_{AR} = ?$

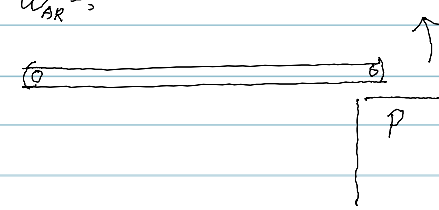
$TE_3 = TE_4$

$\frac{1}{2} I_o \omega_{AR}^2 = 2.6\,ft\cdot lbs$

$\frac{1}{2}(.932)\,\omega_{AR}^2 = 2.6$

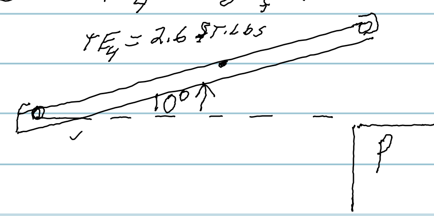
$\omega_{AR} = 2.36\,\frac{rad}{s}$ CCW

③ $TE_3 = \frac{1}{2} I_o \omega_{AR}^2$



④ $TE_4 = mgh_f = (10lb)\,1.5\sin 10$

$TE_4 = 2.6\,ft\cdot lbs$



STEP 3 → $F = ?$

$\Sigma M_o \Delta T = I_o(\omega_{AR} - \omega_b)$

$$\left[-(10lbs)(1.5') + (F)(3')\right](.01s) = (0.932)(+2.36 - (-4.77))$$

$$-.15 + .03F = (.932)(7.13\,\tfrac{rad}{s}) = 6.65$$

$$.03F = 6.80$$

$$F_{AVG} = 227\,lbs.$$

⑤ FBD