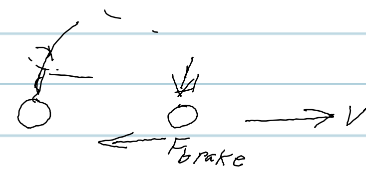
CH. 18 COMBINED TRANSLATION + ROTATION USING FREE-BODY DIAGRAMS

SECTIONS 18-1+2

RIGID BODIES ⇒ $\Sigma F = m a_{CM}$ DYNAMICS $\Sigma M_{CM} = I \alpha$

$\Sigma F = 0$ STATICS ⇒ $\Sigma M = 0$

FBD'S



2-D WORK

① $\Sigma F_x = m a_x$ } AT CM ② $\Sigma M_z = 0 \Rightarrow \Sigma F \cdot r_\perp = 0$

② $\Sigma F_y = m a_y$ ③ $\Sigma M_z = I_z \alpha_z$

3-D

$\Sigma F_x = m a_x$ $\Sigma M_z = I_z \alpha_z$

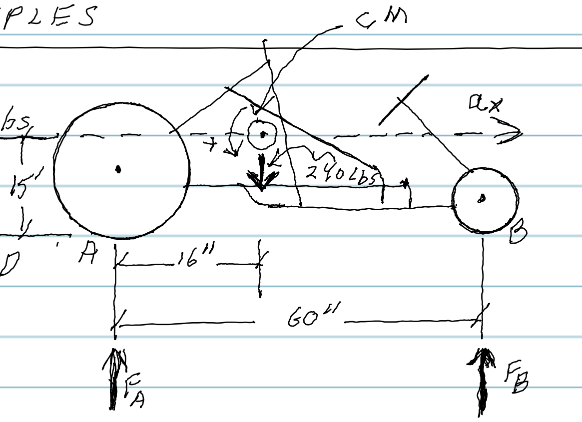
$\Sigma F_y = m a_y$ $\Sigma M_x = I_x \alpha_x$ } 6 EQUATIONS

$\Sigma F_z = m a_z$ $\Sigma M_y = I_y \alpha_y$

!! VIP !! ΣM_{CM} – MUST BE ABOUT THE CENTER OF MASS !!

!! VIP 2 !! $\Sigma F = m a$ ← AT THE C.M.

SECTION 18-1/2 HOMEWORK EXAMPLES



18-6 GIVEN: $F_E = 24$ Lbs, $W = F_g = 240$ Lbs

FIND: $|a| = ?$, $F_A = ?$, $F_B = ?$

SOLUTION: FREE-BODY DIAGRAM - FBD

- 2 TYPES OF FORCES - CONTACT, F_g

a) $\Sigma F_x = m a_x$ ← @ C.M.

$\Sigma F_x = 24$ Lbs $\qquad m = \frac{F_g}{a_g} = \frac{240 \text{ Lbs}}{32.2 \frac{ft}{s^2}} = 7.45$ slugs

$$a_x = |a| = \frac{\Sigma F_x}{m} = \frac{24 \text{ Lbs}}{7.45 \text{ sL}} = \underline{\underline{3.22 \frac{ft}{s^2}}}$$

b) MOMENT MUST BE DONE @ C.M. !! $\qquad \Sigma M_{CM} = I\alpha^{0} = 0$

$\Sigma M_{CM} = \Sigma (F \perp) r_{CM} = 0$

$+ (24 \text{ Lbs}) 15' - (F_A) 16'' \pm (240 \text{ Lbs}) 0 + (F_b)(60'' - 16'') = 0$

① $+360 - 16 F_A + 44 F_b = 0$ {2 UNKNOWNS}

$\Sigma F_y = m a_y$

② $+F_A - 240 + F_B = m/0 = 0 \Rightarrow F_A = 240 - F_b$

② INTO ①

$360 - 16[240 - F_b] + 44 F_b = 0 \qquad \underline{\underline{F_b = 58 \text{ Lbs}}} \qquad$ ② $\underline{\underline{F_A = 182 \text{ Lbs}}}$

LOOK AT WHAT HAPPENED! $\qquad$ STATIC SITUATION! $a_x = 0 \Rightarrow F_E = 0$ Lbs

$\Sigma M_a = 0$ {BECAUSE STATIC}

$(F_A) \cdot 0 - (240 \text{ Lbs}) 16'' + (F_b) 60'' = 0$

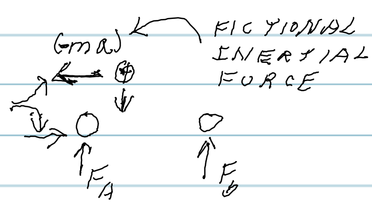
$F_b = \frac{240 \cdot 16''}{60''} = \underline{\underline{64 \text{ Lbs}}}$

SHIFT IN VERTICAL FORCES

WHY YOU MUST!! USE CM FOR MOMENTS.

$\Sigma F = ma$

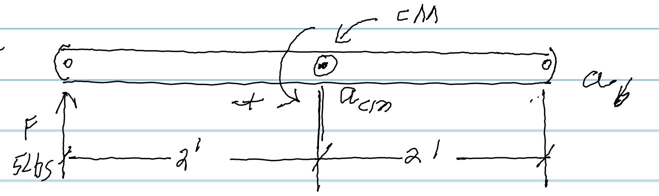
$\underline{\underline{\Sigma F - ma = 0}}$



CH. 18 SECTION 18-1/2 HOMEWORK EXAMPLES (CONT.)

18-10 GIVEN: $F_g = 10\,lbs$, $I_{cm} = 0.8\,sl\cdot ft^2$

FIND: $a_b = ?$



SOLUTION: $\Sigma F = m a_{cm}$

① $a_{cm} = \frac{\Sigma F}{m} = \frac{5\,lbs\,j}{.31\,slug} = 16.1 \frac{ft}{s^2} j$ $\quad F_g = mg \Rightarrow m = \frac{F_g}{g} = \frac{10\,lbs}{32.2\frac{ft}{s^2}} = 0.31\,slugs$

REFER BACK TO SECTION 17-5 Pg. 318 PLANAR MOTION

• $a_a = a_b + a_{a/b} = a_b + \alpha \times r_{a/b} - \omega^2 r_{a/b}$

TRANSLATION (of a_b); ROTATION $\Rightarrow a_T = r\alpha \quad a_{\theta,T} = \frac{v^2}{r} = r\omega^2$

$a_b = a_{cm} + a_{b/cm} = a_{cm} + \alpha_{BAR}(2') - 0^2(2')$

② $a_b = 16.1 j \left(\frac{ft}{s^2}\right) + 2\alpha$ $\quad$ FIND: $\alpha_{BAR} = ?$

③ $\Sigma M_{cm} = I\alpha \Rightarrow \Sigma M_{cm} = ?$

$-F(X) = I\alpha \Rightarrow \alpha = \frac{-(F)X}{I} = \frac{-(5\,lbs)(2\,ft)}{0.8\,sl\cdot ft^2} = -12.5 \frac{rad}{s^2}$

②' $a_b = 16.1 j + \left(-12.5 \frac{rad}{s^2}\right) 2\,ft = 16.1 j - 25 j \; ft/s^2 = -8.9 \frac{ft}{s^2}$

USE CROSS PRODUCTS $\left(\alpha \times r_{b/cm}\right) =$

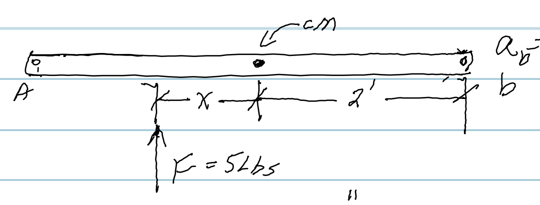
$\alpha = 0 i + 0 j - 12.5 k \left(\frac{rad}{s^2}\right)$

$r = 2 i \; 0 j + 0 k$

$$\begin{vmatrix} i & j & k \\ 0 & 0 & -12.5 \\ 2 & 0 & 0 \end{vmatrix} \begin{matrix} i & j \\ 0 & 0 \\ 2 & 0 \end{matrix} = -25 j \frac{rad}{s^2}$$

ALTERNATE EXTENSION: MUST BE A POINT "X" WHERE $a_b = 0$ (NO KICK)

$a_{cm} = \frac{5\,lbs}{.31\,s} = 16.1 \frac{ft}{s^2}$



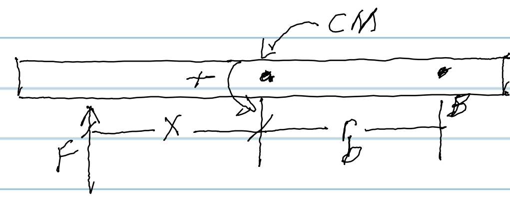
② $a_b = 16.1 j \frac{ft}{s^2} + 2\alpha \Rightarrow$ ③ $\alpha = \frac{-FX}{I}$

$0 = 16.1 j + 2\left[\frac{-FX}{I}\right] = 16.1 j + 2\left[\frac{-5\,lb \cdot X}{0.8\,sl}\right]$

$0 = 16.1 j - 2(6.25)X \Rightarrow X = 1.29\,ft$ "SWEET SPOT" "CENTER OF PERCUSSION"

SECTION 18-2 HOMEWORK EXAMPLE 18-10 EXTENDED

18-10 GENERAL SOLUTION:



$a_{cm} = \frac{F}{m}$, $\Sigma M_{cm} = I\alpha$

$-FX = I\alpha$

$\alpha = \frac{-FX}{I}$

SETTING $a_b = 0$

$a_b = a_{cm} + r\alpha \Rightarrow 0 = a_{cm} + r\alpha$

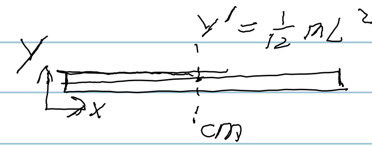
$r\alpha = -a_{cm}$

$r\left[\frac{-FX}{I}\right] = \frac{-F}{m} \Rightarrow X = \frac{I}{rm}$

• X & r_b MUST BE ON OPPOSITE SIDES OF THE CM.

– CENTER OF PERCUSSION –

EXAMPLE: UNIFORM MASS ALONG A LINE: APPENDIX C Pg 605



$I_{y'} = \frac{1}{12} mL^2$

$X = \frac{\frac{1}{12} m (2r)^2}{rm} = \frac{1}{3} r$

18-61 HOMEWORK HELP –

SOLVE PROBLEM: 9 EQUATIONS
9 UNKNOWNS

MATHCAD – HELP – RESOURCE CTR –
QUICK SHEET – GIVEN – FIND

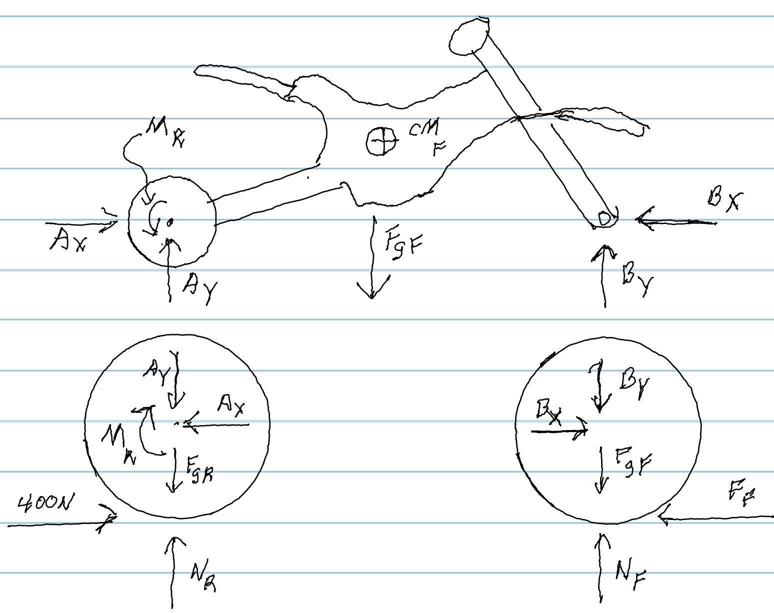
GIVEN {RESERVE WORDS}

9 EQUATION $N_R \Rightarrow N.R$

= ← {SYMBOLIC }

ANS = FIND($N_F, N_R, A_X, A_Y, \ldots$)

ANS = $\begin{pmatrix} N_F \\ N_R \\ A_X \\ A_Y \\ \vdots \end{pmatrix}$



N_F a =