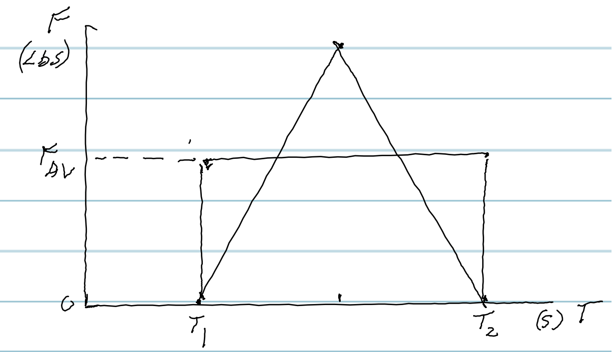
CH. 16 SECTION 16-1 IMPULSE + MOMENTUM → EXAMPLES

$$\Sigma F = ma = m\frac{dV}{dT}$$

$$\underbrace{\int F\,dT}_{\text{IMPULSE}} = \int_{V_1}^{V_2} m\,dV = mV\Big|_{V_1}^{V_2} = mV_2 - mV_1$$

← Lb·s, N·s

IMPULSE



- $$\int F\,dT = mV_2 - mV_1$$

$$\int_{T_1}^{T_2} F\cdot dT = \text{AREA} = F_{AV}(T_2 - T_1)$$

$$F_{AV}(T_2 - T_1) = \int_{T_1}^{T_2} F(T)\,dT$$

- $$F_{AV} = \frac{1}{(T_2 - T_1)}\int_{T_1}^{T_2} F(T)\,dT$$

16-1 GIVEN: $m = 20\,\text{Kg}$, $T = 0$, $V = 0$, $F_x = 10 + 2T^2$ (N) ← SEC

FIND: IMPULSE = ?

a) SOLUTION: $\text{IMPULSE} = \int_0^4 F(T)\,dT = \int_0^4 (10 + 2T^2)\,dT = 10T + \frac{2}{3}T^3\Big|_0^4 = \underline{\underline{82.7\ \text{N·s}}}$

b) $V_2 = ?$ @ $T = 4$ sec

SOLUTION: $\text{IMPULSE} = mV_2 - \cancel{mV_1}^{\,0} \Rightarrow 82.7\,\text{N·s} = 20\,\text{Kg}\,V_2$

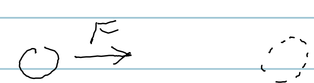
$V_2 = \frac{82.7\,\text{N·s}}{20\,\text{Kg}} = \underline{\underline{4.13\ \text{m/s}}}$

16-27 RAIL GUN GIVEN: $m = 30\,g = .030\,\text{Kg}$, $V_1 = 0$, $V_2 = 5\frac{\text{km}}{s} = 5\cdot 10^3\frac{m}{s}$

$F = 830(1 - .08T)$ (N)

FIND: $F_{AV} = ?$

$\Delta T = 0.0004\,s$



$V_1 = 0$ $\quad V_2 = 5\cdot 10^3\,\frac{m}{s}$

SOLUTION: $F_{AV}(T_2 - T_1) = \int_0^{.0004} F(T)\,dT = mV_2 - mV_1$

$$F_{AV} = \frac{mV_2 - mV_1}{\Delta T} = \frac{(.030\,\text{Kg})\,5\cdot 10^3\,\frac{m}{s} - 0}{.0004\,s} = \underline{\underline{3.75\cdot 10^5\,N = 375.\,\text{kN}}}$$

CH. 16 SECTION 16-2 IMPACTS + CENTER OF MASS

EXTERNAL FORCES

$$V_{cm} = \frac{m_a V_A + m_b V_b}{m_a + m_b}$$

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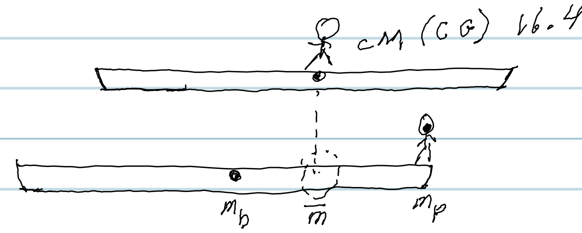
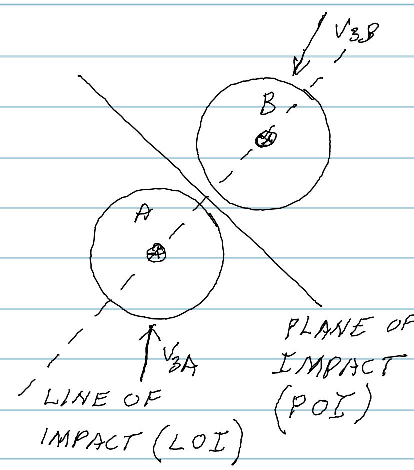


CLASSIFICATION OF IMPACT TYPES



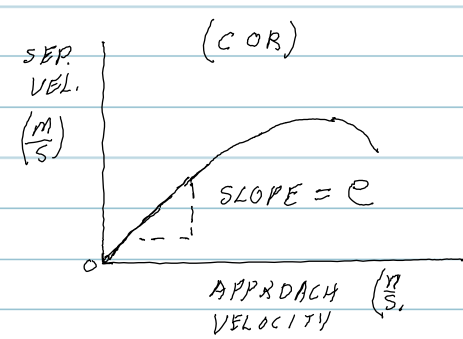
	PRE-IMPACT VELOCITY	CENTER OF MASS	IMPACT TYPE
1)	PARALLEL TO LOI	ON LOI	DIRECT (1-D) CENTRAL
2)	// TO LOI	NOT ON LOI	DIRECT (CAUSES ROTATION) ECCENTRIC
3)	NOT // TO LOI	ON LOI	OBLIQUE (2-D - NO ROTATION) CENTRAL
4)	NOT // TO LOI	NO ON LOI	OBLIQUE (2-D - CAUSE ROTATION) ECCENTRIC

$$e = \frac{\text{SEPERATION VELOCITY}}{\text{APPROACH VELOCITY}} = \frac{V_B' - V_A'}{V_A - V_b} = \frac{V_{2B} - V_{2A}}{V_{1A} - V_{1B}}$$

NO UNITS

COEFFICIENT OF RESTITUTION (COR)

	TYPE OF IMPACT
$e = 0$	PLASTIC
$0 < e < 1$	ELASTIC
$e = 1$	PERFECTLY ELASTIC $TKE_b = TKE_A$



SECTION 16-2 NOTES (CONT.)

1-D MOTION (CENTRAL IMPACT)

$$m_a V_A + m_b V_b = m_a V_a' + m_b V_b'$$

- $$V_{2A} = \frac{V_{1A}(m_a - m_b e) + m_b V_{1b}(1+e)}{(m_a + m_b)}$$

$$e = \frac{V_b' - V_a'}{V_a - V_b}$$

IMPUTS: $V_{1a} = ?$ $m_a = ?$
$V_{1B} = ?$ $m_b = ?$
$e = ?$

- $$V_{2B} = \frac{m_a V_{1A}(1+e) + V_{1B}(m_b - m_a e)}{(m_a + m_b)}$$

NOTE: USE MATHCAD – CH16_COR_IMPACT.mcd

- TWO-DIMENSIONAL MOTION – OBLIQUE CENTRAL IMPACT

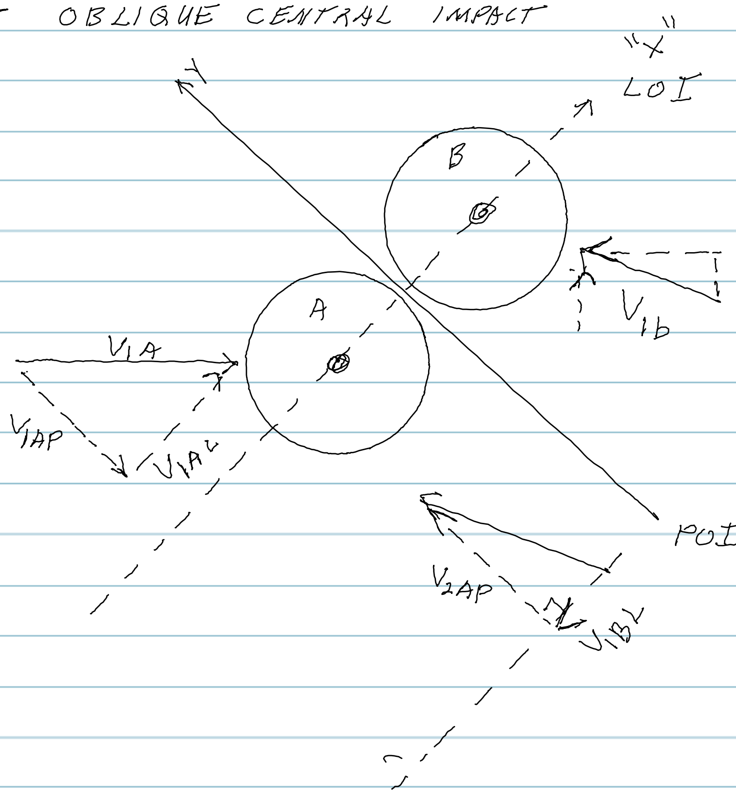
GOOD NEWS!!

$$V_{2AL} = \frac{V_{1AL}(m_a - m_b e) + m_b V_{1bL}(1+e)}{(m_a + m_b)}$$

$$V_{2bL} = \frac{m_a V_{1AL}(1+e) + V_{1bL}(m_b - m_a e)}{(m_a + m_b)}$$

$\cancel{V_{2AP} = V_{2AL} \; ; \; V_{2bP} = V_2}$

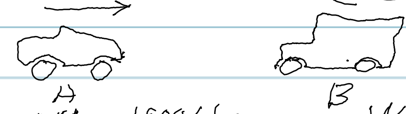
$V_{2AP} = V_{1AP}$, $V_{2bP} = V_{1bP}$



SECTION 16-2 HOMEWORK EXAMPLE MOMENTUM

16-61 LINEAR EQUATION: $e = 0.15$

FIND: $V_{2A} = ?$ $V_{2B} = ?$



SOLUTION: 1-D (CENTRAL IMPACT)

$$V_{2A} = \frac{V_{1a}(m_a - m_b e) + m_b V_{1b}(1+e)}{(m_a + m_b)} = \frac{30(1900 - 2800(0.15)) + 2800(-20)(1+.15)}{(1900 + 2800)}$$

VEP

$$V_{2A} = \cancel{-6.} -4.26 \text{ mph} = \underline{\underline{-6.24 \tfrac{ft}{s}}}$$

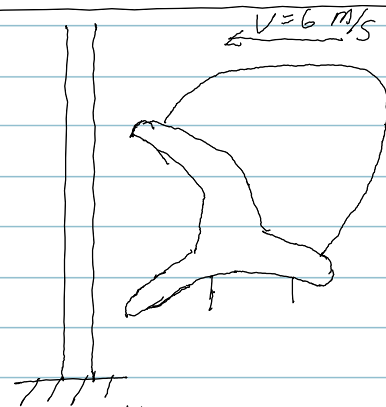
$$V_{2b} = \frac{m_a V_{1a}(1+e) + V_{1B}(m_b - m_a e)}{(m_a + m_b)} = \frac{1900(30)(1+.15) + (-20)(2800 - 1900(.15))}{(1900 + 2800)}$$

$$V_{2b} = \underline{\underline{3.24 \text{ mph} = 4.76 \tfrac{ft}{s}}}$$

16-74 HELMET DESIGN

$m_H = 2.4$ Kg
$m_p = 2$ Kg
$e_{H-S} = .85$

FIND: $V_p = ?$



SOLUTION:

$e_{p-H} = .15$

$$e_{H-S} = .85 = \frac{\text{SEPER. V}}{\text{APPROACH V}} = \frac{\text{SEP. V}}{6\ m/s}$$

$$\text{SEP. } V_H = .85\left(6 \tfrac{m}{s}\right) = 5.1 \tfrac{m}{s}$$

$V_a = 5.1$ m/s → $\boxed{H}$ A
$\leftarrow 6$ m/s $- V_B$ $\boxed{PER}$ B
$m_a = 2.4$ kg $m_b = 2$ Kg.

VELOCITY HEAD AFTER REBOUND

$$\underline{\underline{V_{2b} = 0.96 \tfrac{m}{s}}}$$

$V_{1a} = +5.1$ m/s, $m_a = 2.4$ Kg
$V_{1b} = -6.$ m/s, $m_b = 2$ Kg
$e = .15$

SECTION 16-2 HOMEWORK EXAMPLES (CONT.)

16-79 2-D IMPACT

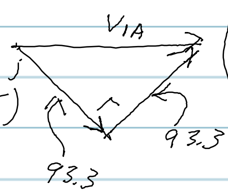
GIVEN: $V_{1A} = 132(\cos(45)\,i + \cos(45)\,j)$ $\left(\frac{FT}{S}\right)$

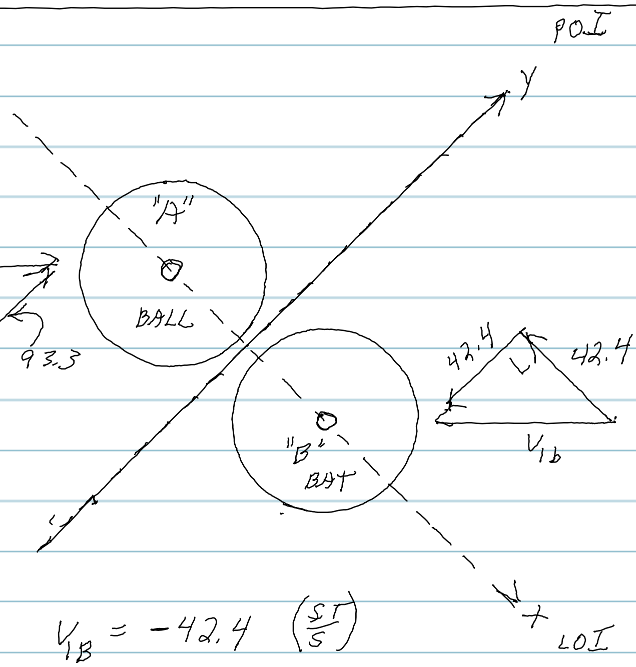
$V_{1A} = 93.34\,i + 93.34\,j$ $\frac{FT}{S}$

COR $e = .2$

$V_{1b} = -60\cos(45)\,i - 60\cos(45)\,j$

$V_{1b} = -42.43\,i - 42.43\,j$ $\left(\frac{FT}{S}\right)$





NEGLECT: $V_{1b} = V_{2b} \Rightarrow m_b >> m_{ball} = m_a$

a) FIND $V_{2a} = ?$

SOLUTION:

1-D ALONG "LOI"

$V_{1A} = +93.34$ $\frac{FT}{S}$ $\quad V_{1B} = -42.4$ $\left(\frac{FT}{S}\right)$

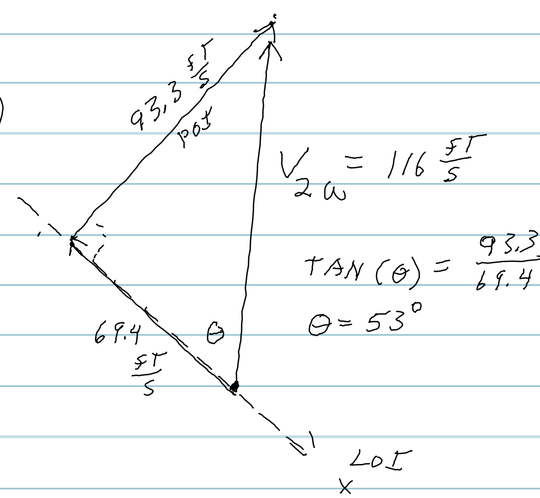
$m_a = 1$ $\quad m_b = 1000$

$e = .2$

$V_{2a} = -69.4$ $\frac{FT}{S}$ (ALONG LOI AXIS)

$\overline{V_{2a}} = -69.4\,i + 93.3\,j$ $\left(\frac{FT}{S}\right)$

b) FOUL BALL? FOUL



$\tan(\theta) = \frac{93.3}{69.4}$

$\theta = 53°$

SECTION 16 - 4 MASS FLOWS (MOMENTUM ENGINES)

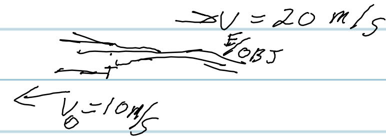
$$F_{MF} = ma = m\frac{\Delta V}{T} = \left(\frac{m}{T}\right)\Delta V$$

MASS FLOW RATE: $\left(\frac{m}{T}\right)$ — $\left(\frac{kg}{s}\right), \frac{slugs}{s}$

EXIT VELOCITY (RELATIVE VELOCITY): ΔV — $\frac{m}{s}, \frac{ft}{s}$

$V_{EXIT/OBJ}$

F_{MF}: N, lbs



DETAILS

- $F_{MF} = -\frac{\dot{m}}{T}\Delta V$
- INPUT FLOW

$$\Delta V_{in} = (V_0 - V_{in})$$

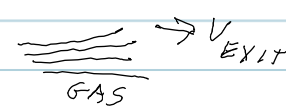
$$\Sigma F_{MF} = \left(F_{MF_{in}} - F_{out}\right)$$

- MASS FLOW - GENERAL - CONSTANT $\quad F_{MF} = \frac{dm(T)}{dT} \cdot V_{EX/OB}(T)$

- INTERESTING DETAIL - FIND AN ENGINEER

TYPES OF ENGINES	MASS FLOW
JET	YES
PROP'S	YES
BOATS (PROP'S)	YES
CARS	YES (PUSH - EARTH)

- EFFICIENCY OF MASS FLOW ENGINES



$$F_{MF} = \frac{m}{T}\Delta V \Rightarrow \frac{2M}{T}\left(\frac{1}{2}\Delta V\right) \qquad KE_{V_{EXT}} = \frac{1}{2}m(\Delta V)^2$$

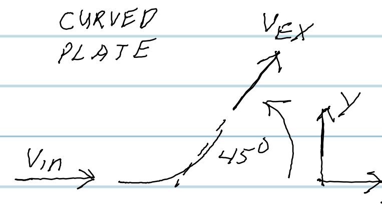
$$KE_{EXIT} = \frac{1}{2}(2M)\left(\frac{1}{2}\Delta V\right)^2 = \frac{1}{2}\left(\frac{1}{2}m\Delta V^2\right)$$

SECTION 16-4 HOMEWORK EXAMPLE

16-104 DEFLECTION OF MASS FLOW

GIVEN: $MFR = \frac{30\,kg}{s}$ $V_1 = 40\,m/s$, $\frac{dm}{dT} = 30\,\frac{kg}{s}$

FIND: $F = ?$ (ACTING ON PLATE)



SOLUTION: $\Sigma F = -\frac{dm}{dT}\Delta V_{IN} - \frac{dm}{dT}(\Delta V_{EX})$

a) $\Sigma F = \left(-\frac{dm}{dT}\right)(\Delta V_{in} + V V_{EX})$

$V_{in_1} = 40\,i \left(\frac{m}{s}\right)$

$V_{EX2} = 40\,\frac{m}{s}(\sin(45))i + \cos$

$V_{EX2} = 28.28\,i + 28.28\,j \left(\frac{m}{s}\right)$

$V_{in2} = 0\,\frac{m}{s}$

$V_{EX1} = 0$

$\Delta V_{in} = V_{in2} - V_{in1} = 0 - 40\,i$

$\Delta V_{in} = -40\,i \left(\frac{m}{s}\right)$

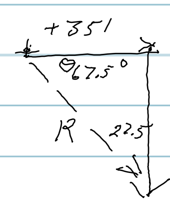
$\Delta V_{EX} = V_{EX2} - V_{EX1}$

$\Delta V_{EX} = 28.28\,i + 28.28\,j - 0$

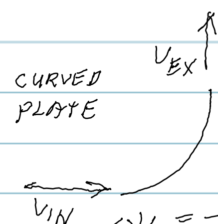
$\Delta V_{EX} = 28.28\,i + 28.28\,j\ \frac{m}{s}$

$\Sigma F = \left(-30\,\frac{kg}{s}\right)(-40\,i + 28.28\,i + 28.28\,j)$

$\Sigma F = +351\,i - 848\,j\ (N)$ FORCE ACTS ON PLATE!!



b) $\Sigma F = \left(-\frac{dm}{dt}\right)(\Delta V_{in} + \Delta V_{EX})$



$\Sigma F = \left(-30\,\frac{kg}{s}\right)(-40\,i + 40\,j)$

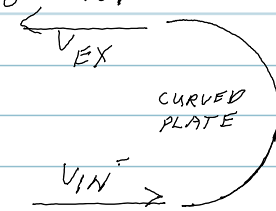
$\Sigma F = +1200\,i - 1200\,j\ (N)$

$\Delta V_{in} = -40\,i$

$\Delta V_{EX} = +40\,j + 0\,i$

c) $\Sigma F \left(-\frac{dm}{dt}\right)(\Delta V_{in} + V_{EX})$

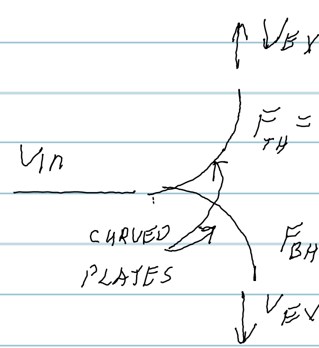
$\Delta V_{EX} = -40\,i + 0 = -40\,i$



$\Sigma F = \left(-30\,\frac{kg}{s}\right)(-40\,i - 40\,i)$

$\Sigma F = +2400\,i\ N$

$\Delta V_{in} = -40\,i$



$F_{TH} = +600\,i - 600\,j\ (N)$

$F_{BH} = 600\,i + 600\,j\ (N)$

$F_{TOT} = 1200\,i + 0\,j\ (N)$