CHAPTER 8 SECTION 8.1 MOMENTS OF INERTIA

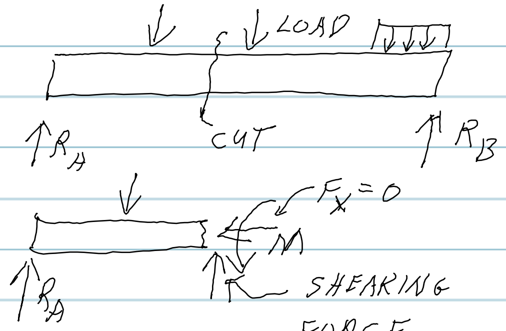
INTRODUCTION: PHYSICS $F = ma$

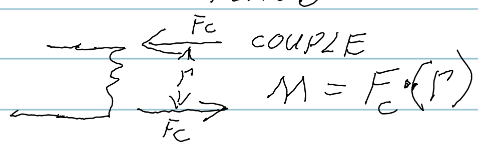
$M = T = I\alpha$ ← ANG. ACCEL. $\frac{rad}{s^2}$

MOMENT — MASS MOMENT OF INERTIA

AREA ⇒ MOMENT OF INERTIA

BEAM





PROPERTY SPAN, LOADS

$$\sigma = \frac{Mc}{I}$$ FLEXURE FORMULA

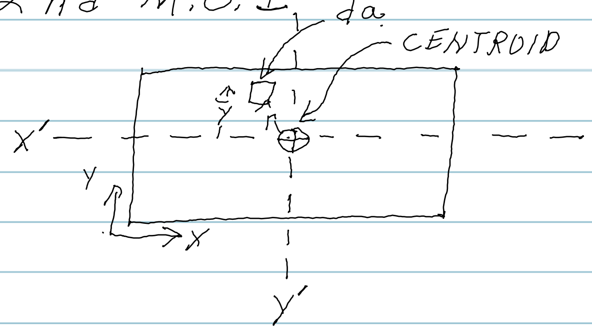
σ: Pa, PSI — MATERIAL CHOICE ONLY

I: SHAPE (AREA DIST) OF THE BEAM – STRENGTH OF THE SHAPE OF THE MATERIAL

MOMENT OF INERTIA – 2nd M.O.I.

$$I_x = \int \bar{y}^2 \, dA$$

$$I_y = \int \bar{x}^2 \, da$$



$$J_0 = \int \bar{r}^2 \, da = I_x + I_y$$

STRENGTH OF THE AREA AGAINST TWISTING

$$I_{xy} = \int \bar{x}\bar{y} \, da \Rightarrow \text{PRODUCT OF M.O.I.}$$

$$I_y = K_y^2 A$$ ← CROSS SECTIONAL AREA

RADIUS OF GYRATION

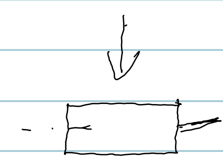
$$I_x = k_x^2 A$$

$$J_0 = K_0^2 A \qquad K_0^2 = K_x^2 + K_y^2$$

Pg 577 $I_{x'} = \frac{1}{12} b h^3$

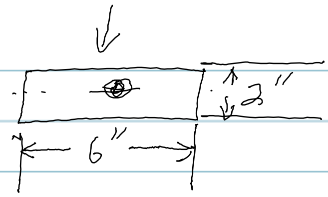
STRENGTH AGST BENDING AROUND THE "X" AXIS

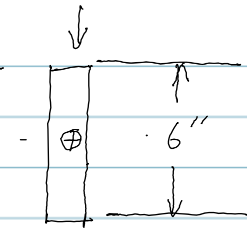
I_x



$I_{y'} = \frac{1}{12} h b^3$

SECTION 8.1 (CONT)





$$I = \frac{1}{12} bh^3 = \frac{1}{12}(6)2^3 = \cancel{24}\ 4\ in^4$$

$$I_{x'} = \frac{1}{12} bh^3 = \frac{1}{12}(2)(6)^3 = 36\ in^4$$

$$\underset{psi}{\sigma} = \frac{Mc}{I}$$

NOTE ON RELATIONSHIP BETWEEN "I" & SHAPE

$F = ma$

$M = Fr$

$a = r\alpha$

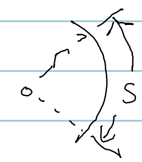
$s = r\theta$ (ARC LENGTH; rad)

$C = r(2\pi)$

$\frac{s}{T} = \frac{r\theta}{T}$

$v = r\omega$ (LINEAR V.; ANGULAR VEL.)

$a = r\alpha$



$\left(\frac{M}{r}\right) = mr\alpha$

$M = mr^2\alpha$

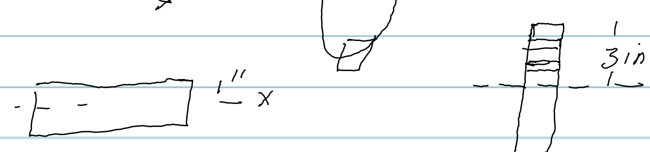
DEFINE: $I = mr^2$

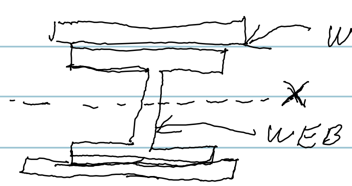
$s = r\theta$

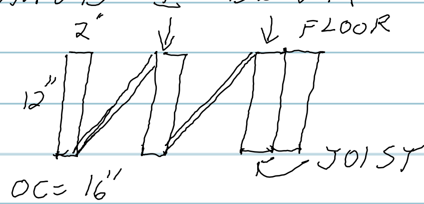
$v = r\omega$

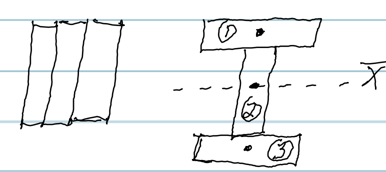
$a_T = r\alpha$

$I_x = \int y^2\, da$

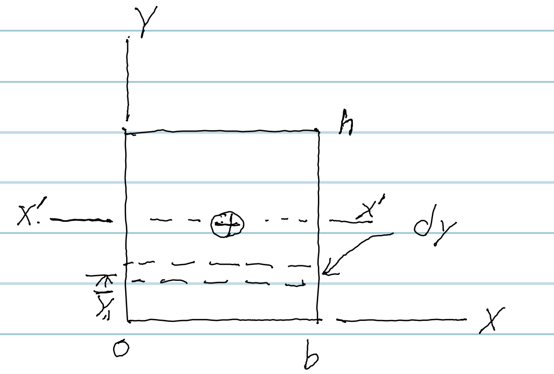








SECTION 8.1 MOMENTS OF INERTIA EXAMPLE



GIVEN:

FIND: $I_x = ?$

SOLUTION: $I_x = \int \bar{y}^2 \, da$

$$I_x = \int_0^h y^2 b \, dy = b\left(\frac{y^3}{3}\right)\Big|_0^h$$

$da = b\,dy$

$$\underline{\underline{I_x = \frac{1}{3} b h^3}}$$

FIND: $I_{x'} = ?$

SOLUTION: $I_{x'} = \int \bar{y}^2 \, da = \int_{-\frac{1}{2}h}^{+\frac{1}{2}h} y^2 (b\,da) = b\frac{y^3}{3}\Big|_{-\frac{1}{2}h}^{+\frac{1}{2}h}$

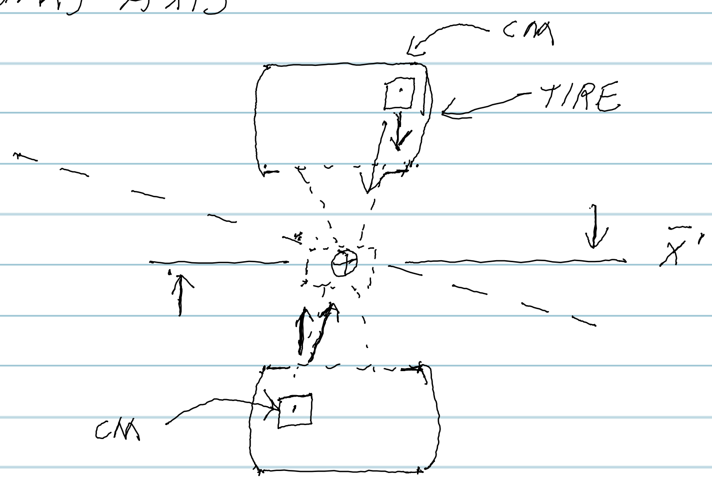
$$I_{x'} = b\frac{(\frac{1}{2}h)^3}{3} - \frac{b(-\frac{1}{2}h)^3}{3} = \frac{1}{24}bh^3 + \frac{1}{24}bh^3$$

$\underline{\underline{I_{x'} = \frac{1}{12}bh^3}}$ NOTE: $I_{x'}$ IS SMALLER

V.I.P. $I_{x'}$ IS A MINIMUM

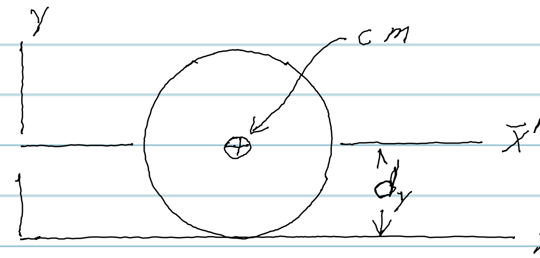
1) OBJECT WILL SPIN ABOUT ITS C.M.
2) SPIN ABOUT I (MINIMUM) AXIS

3) FINAL NOTE



SECTION 8.2 MOMENTS OF INERTIA – COMPOSITE SHAPES (PARALLEL – AXIS TRANSFER EQUATI

FINDING I AROUND A DIFFERENT AXIS



Pg 61 –

$$\bar{x} = \frac{\int_A x'\,dA}{\int_A dA} = 0$$

CENTROID

$$\Rightarrow \int_A x'\,dA = 0$$

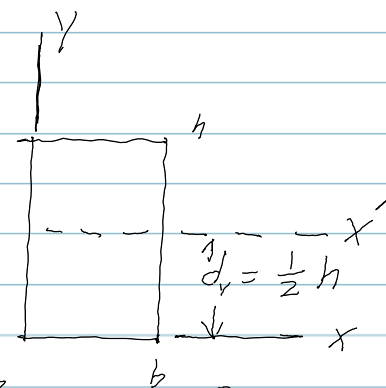
$$I_x = \int_A y^2\,dA = \int (y' + d_y)^2\,dA = \int (y'^2 + 2d_y y' + d_y^2)\,dA$$

$$I_y = \int y'^2\,dA + 2d_y \int y'\,dA + d_y^2 \int dA$$

$$I_y = I_{x'} + 0 + d_y^2 \cdot A$$

$$I_x = I_{x'} + d_y^2 \cdot A_{OBJECT}$$

CENTROIDAL AXIS



$$I_{x'} = \frac{1}{12} bh^3$$

$$I_x = \frac{1}{12} bh^3 + \left(\frac{1}{2}h\right)^2 [bh]$$

$$I_x = \frac{1}{12} bh^3 + \frac{1}{4} bh^3 = \frac{4}{12} bh^3 = \frac{1}{\cancel{4}_3} bh^3 = \frac{1}{3} bh^3$$

$$I_x = I_{x'} + d^2 A$$

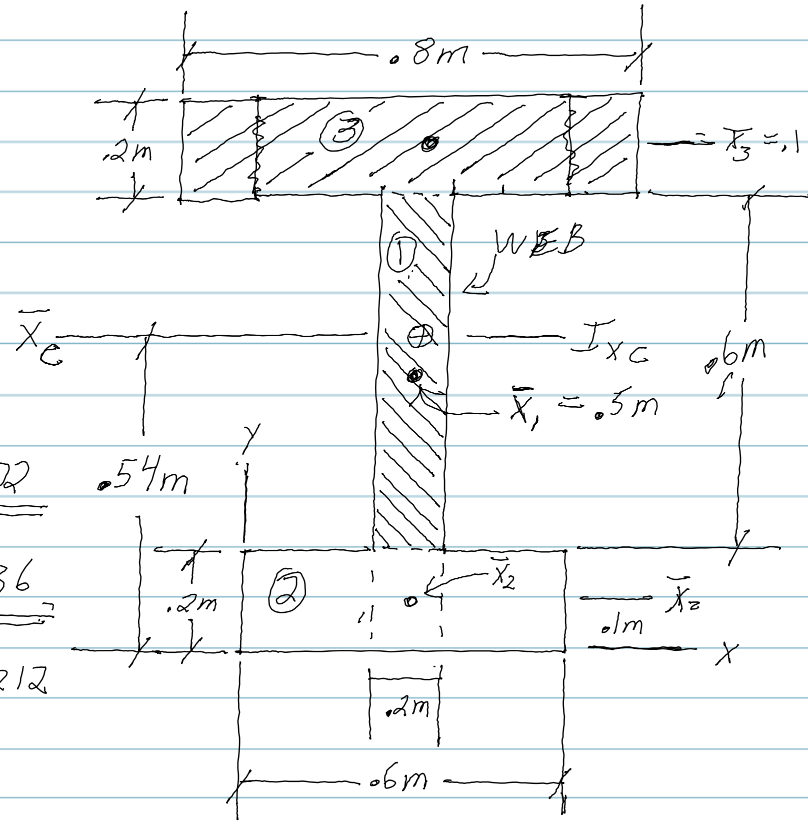
ANY AXIS — CENTROID ONLY

V.I.P. NOTE: $I_{x'} \leftarrow$ CENTROID $\quad I_x \leftarrow$ ANY OTHER ∥ AXIS

SECTION 8.2 COMPOSITE MOMENT OF INERTIA PROBLEM

PROB. 8-31

GIVEN:



FIND: I_{XC}

PLAN: $I_{XC1} = I_{X1} + Ad^2$

$A_1 = (.2m)(.6m) = .12\ m^2$

$A_2 = (.6m)(.2m) = .12\ m^2$

$A_3 = (.2m)(.8m) = .16\ m^2$

$d_1 = \bar{X}_c - \bar{X}_1 = .54 - .5 = .04$

$d_2 = \bar{X}_c - \bar{X}_2 = .54 - .1 = .44$

$d_3 = \bar{X}_c - \bar{X}_3 = .54 - .9 = -.36$

$I_{X1} = \frac{1}{12} bh^3 = \frac{1}{12}(.2)(.6)^3 = 3.6 \cdot 10^{-3}$

$I_{X2} = \frac{1}{12}(.6)(.2)^3 = 4 \cdot 10^{-4}$

$I_{X3} = \frac{1}{12}(.8)(.2)^3 = 5.33 \cdot 10^{-4}$

$I_{XC1} = 3.6 \cdot 10^{-3} + (.12)(.04)^2 = .00372$

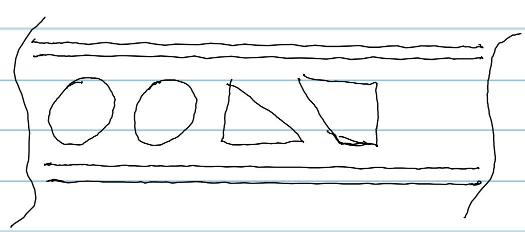
$I_{XC2} = 4 \cdot 10^{-4} + (.12)(.44)^2 = .0236$

$I_{XC3} = 5.33 \cdot 10^{-4} + (.16)(-.36)^2 = .0212$

$I_{XC} = I_{XC1} + I_{XC2} + I_{XC3}$

$I_{XC} = .00372 + .0236 + .0212$

$I_{XC} = .0482\ m^4$



$K_X = \sqrt{\frac{I_X}{A}} = \sqrt{\frac{.0482\ m^4}{(.12 + .12 + .16)\ m^2}}$

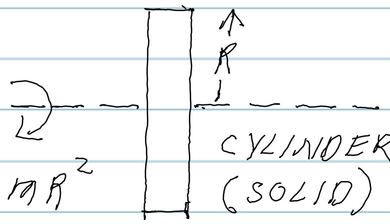
$K_X = .35\ m$

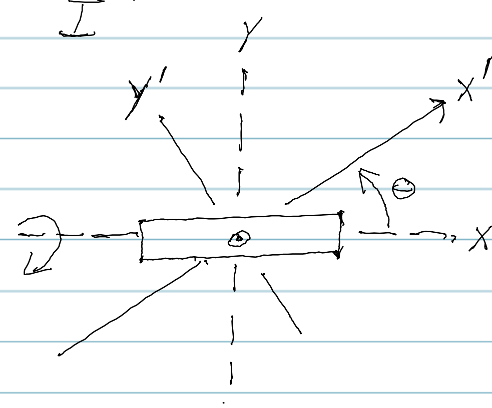
SECTION 8.3 ROTATED & PRINCIPAL AXES – MAXIMUM MOMENT OF INERTIA

FINDING THE MAXIMUM + MINIMUM "I"
(SEE Pg 61)

$I = \sum m r^2$

$I = \frac{1}{4} m R^2$





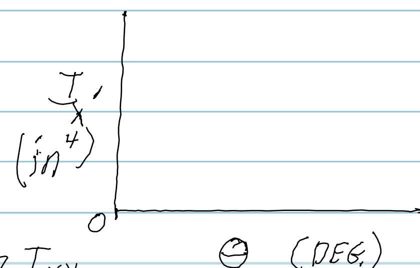
NEED: I_x, I_y, I_{xy}

$$I_{x'} = I_x \cos^2(\theta) - 2 I_{xy} \sin(\theta)\cos(\theta) + I_y \sin^2(\theta)$$

$$I_{y'} = I_x \sin^2(\theta) + 2 I_{xy} \sin(\theta)\cos(\theta) + I_y \cos^2(\theta)$$

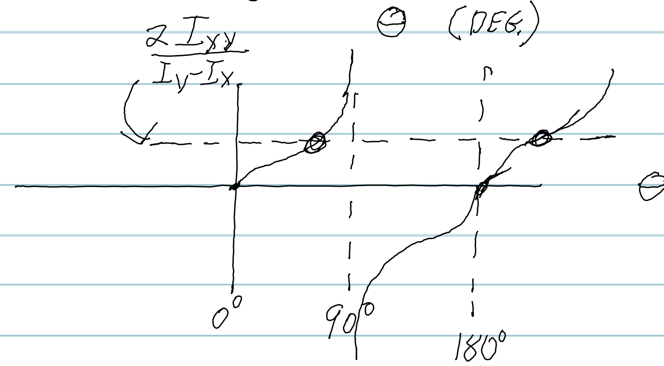
$$I_{x'y'} = (I_x - I_y)\sin(\theta)\cos(\theta) + (\cos^2(\theta) - \sin^2(\theta)) I_{xy}$$

TO FIND MAX. OR MIN. I'
(PRINCIPAL AXES)

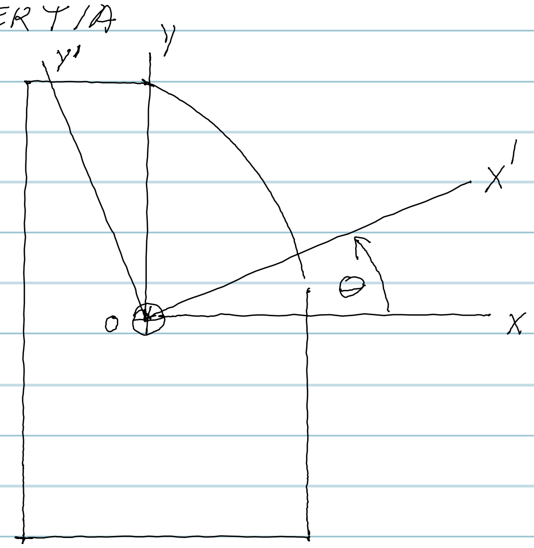


SET: $\frac{d(I_{x'})}{d\theta} = 0$

$$\tan(2\theta_p) = \frac{2 I_{xy}}{I_y - I_x}$$



SECTION 8.3 PROB. 8-91 FIND PRINCIPAL MOMENTS OF INERTIA



GIVEN: $I_x = 1.26 \cdot 10^6 \ in^4$

$I_y = 6.56 \cdot 10^5 \ in^4$

$I_{xy} = -1.02 \cdot 10^5 \ in^4$

FIND: PRINCIPLE AXES & MAX & MIN I

SOLUTION: $\tan(2\theta_p) = \dfrac{2 I_{xy}}{I_y - I_x}$

$$\tan(2\theta_p) = \frac{2(-1.02 \cdot 10^5)}{(6.56 \cdot 10^5 - 1.26 \cdot 10^6)}$$

$$2\theta_p = 18.6^\circ$$

$$\theta = 9.3^\circ \qquad I_{x'} = I_x \cos^2(\theta) - 2 I_{xy} \sin(\theta)\cos(\theta) + I_y \sin^2(\theta)$$

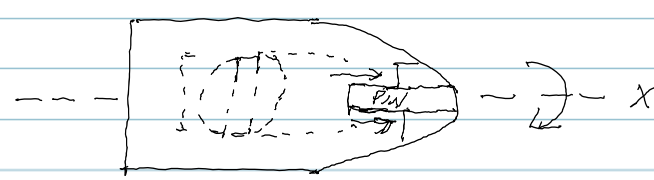
$$I_x' = 1.26 \cdot 10^6 \cos^2(9.3^\circ) - 2(-1.02 \cdot 10^5)\sin(9.3)\cos(9.3) + (1.26 \cdot 10^6)\sin^2(9.3)$$

$$\underline{\underline{I_x' = 1.28 \cdot 10^6 \ in^4}}$$

$$I_y' = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}(\cos(2\theta_p) + I_{xy}\sin(2\theta)$$

$$\underline{\underline{I_y' = 6.38 \cdot 10^5 \ in^4}}$$

APPLICATION:



SECTION 8.4 MOHR'S CIRCLE

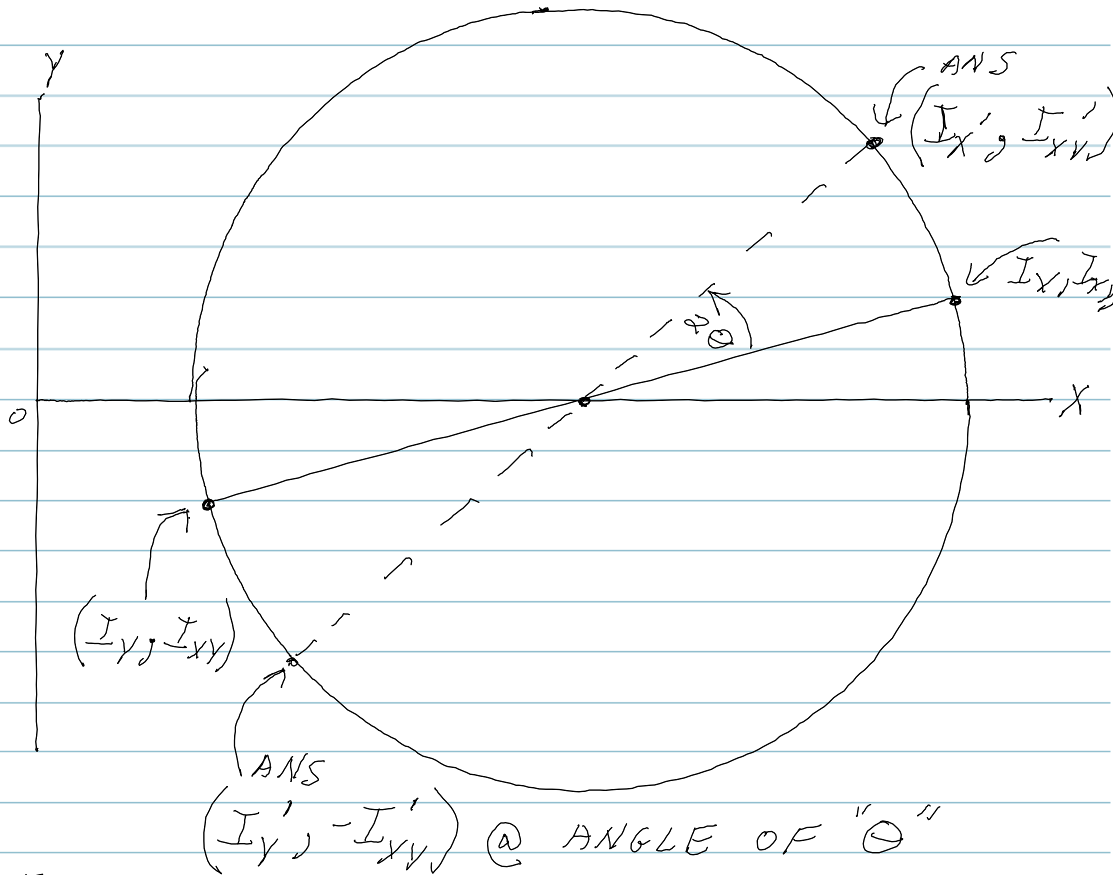
GRAPHICAL SOLUTION TO FIND:
I_x', I_y', I_{xy}'
θ_p

1) GRAPH – EQUAL AXIS

2) PLOT POINTS
(I_x, I_{xy})
$(I_y, -I_{xy})$

3) CONNECT POINTS

4) DRAW CIRCLE



NOTE: $I_{xy} = 0$ @ PRINCIPLE AXIS

SECTION 8.5 + 8.6 MOMENTS OF INERTIA FOR MASSES

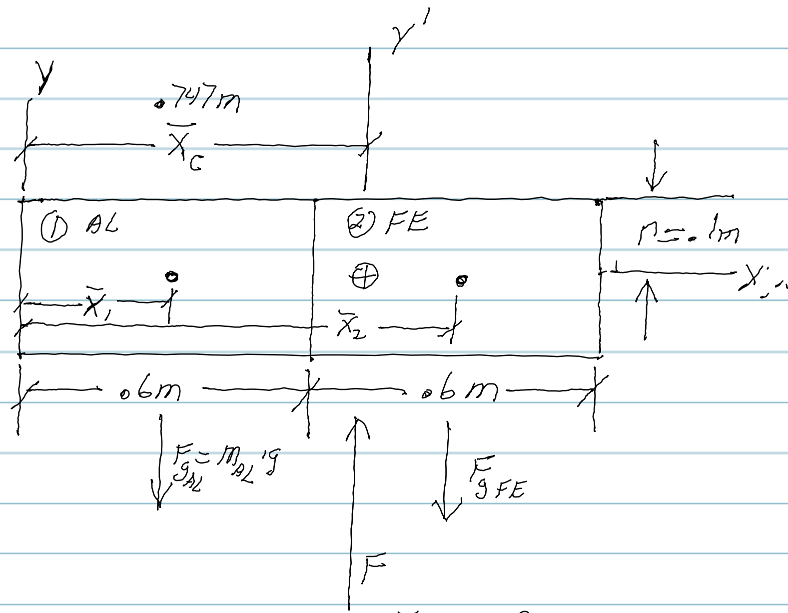
$I_{z\,axis} = I_x + I_y$ (THIN PLATE)

SECTION 8.6 MASS MOMENT OF INERTIA PROB. 8-130

GIVEN: $\rho_{AL} = 2700\ kg/m^3$

$\rho_{FE} = 7860\ kg/m^3$

FIND: $I_{x'}$, $I_{y'}$



$\bar{x}_1 = .3m$ $\quad$ $\bar{x}_2 = .9m$

STEP 1) FIND $\bar{x}_c = ?$

$$\rho_{AL} = \frac{m_{AL}}{V_{AL}}$$

$$m_{AL} = (\rho_{AL}) V_{AL} = \rho_{AL}(\pi r^2 h)$$

$$m_{AL} = \left(2700 \frac{kg}{m^3}\right) \pi (.1m)^2 (.6m)$$

$$m_{AL} = 50.9\ kg$$

$$m_{FE} = \left(7860 \frac{kg}{m^3}\right) \pi (.1m)^2 (.6m) = 148\ kg$$

REMEMBER $F = mg$

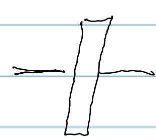
$$\bar{x}_c = \frac{\Sigma m_i \bar{x}_i}{\Sigma m_i} = \frac{(50.9kg)(.3) + (148kg)(.9m)}{50.9 + 148\ kg} = \underline{\underline{.747m}}$$

STEP 2) $I_{x'} = ?$ $\quad$ EASIER $\Rightarrow$ ON SKETCH

DON'T NEED PARALLEX AXIS TRASFER

$$I_{xc} = I_{x1} + I_{x2} \qquad I_{xc} = I_{x1} + md^2 \Rightarrow d = 0$$

THIN CIRCULAR PLATE $\quad I_{z'} = \frac{1}{2} m R^2$



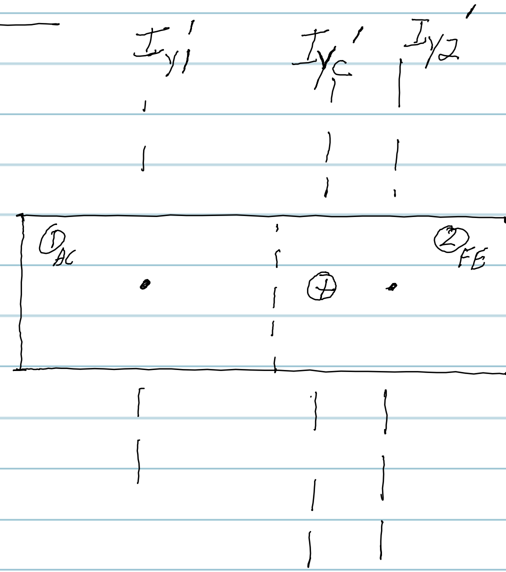
CIRCULAR CYLINDER $\quad I_{z'} = \frac{1}{2} m R^2$

$$\left.\begin{aligned} I_{x1} &= \frac{1}{2} m_{AL} R^2 = \frac{1}{2}(50.9kg)(.1m)^2 = \\ I_{x2} &= \frac{1}{2} m_{AL} R^2 = \frac{1}{2}(148kg)(.1m)^2 = \end{aligned}\right\} 1.\ kg/m^3$$

$$\underline{\underline{I_{xc} = I_{x1} + I_{x2} = 1.\ kg/m^3}}$$

Prob. 8-130 (CONT.)

CIRCULAR CYLINDER



$$I_{y1}' = m_{AC}\left(\tfrac{1}{12}L^2 + \tfrac{1}{4}R^2\right)$$

$$I_{y1}' = (50.9\,kg)\left(\tfrac{1}{12}(.6)^2 + \tfrac{1}{4}(.1)^2\right)$$

$$I_{y1}' = 1.654\ kg \cdot m^2$$

$$I_{y2}' = (148)\left[\tfrac{1}{12}(.6)^2 + \tfrac{1}{4}(.1)^2\right]$$

$$I_{y2}' = 4.81\ kg \cdot m^2$$

SHAPE	$\bar{X}_i$ (m)	MASS (kg)	I_{yi}' (kg·m²)	d_i $(\bar{X}_c - \bar{X}_i)$	I_{yci}'
1	.3	50.9	1.654	.447	11.82
2	.9	148.	4.81	-.153	8.27

$$I_{yc1}' = I_{y1}' + m_1 d_1^2 = 1.654 + (50.9)(.447)^2 = 11.82\ kg \cdot m^2$$

$$I_{yc2} = 4.81 + (148)(-.153)^2 = 8.27\ kg \cdot m^2$$

FINALY:

$$I_{yc}' = I_{yc1} + I_{yc2} = 11.82 + 8.27 = \underline{\underline{20.1\ kg \cdot m^2}}$$