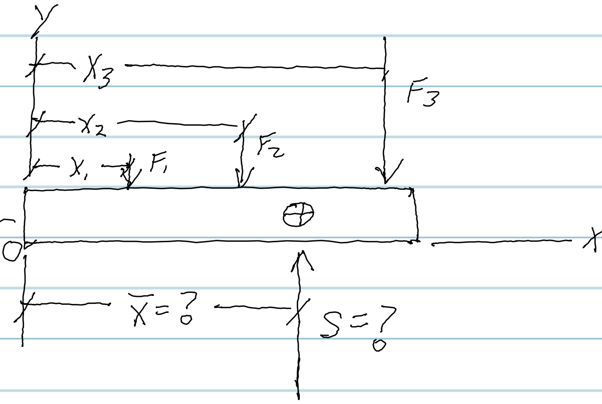
CH. 7 CENTROIDS & CENTERS OF MASS

SECTION 7.1

CTR OF MASS (C.G.) FBD



$\Sigma M_O = 0$

$-x_1 F_1 - x_2 F_2 - x_3 F_3 + \bar{x} S = 0 \quad +\circlearrowleft$

$\bar{x} S = x_1 F_1 + x_2 F_2 + x_3 F_3$

$\bar{x} = \dfrac{x_1 F_1 + x_2 F_2 + x_3 F_3}{S}$

$\Sigma F_y = 0$

$-F_1 - F_2 - F_3 + S = 0$

$\underline{\underline{S = F_1 + F_2 + F_3}}$

$\bar{x} = \dfrac{x_1 F_1 + x_2 F_2 + x_3 F_3 + \ldots}{F_1 + F_2 + F_3 + \ldots}$

$\bar{x} = \dfrac{\sum_{n=1}^{n} x_i F_i}{\sum_{n=1}^{n} F_i}$ EQUATION OF C.O.M. SHORTCUT FOR USE A FBD

WEIGHT MASS

$F = ma \Rightarrow F = mg$ $\qquad F \cong m$

PROPORTIONAL TO "F"

1) MASS (kg, SLUGS $\Rightarrow m = \dfrac{F}{g} \Rightarrow m_{\text{SLUGS}} = \dfrac{F \text{ LBS}}{32.2 \text{ FT}/S^2}$

2) VOLUME (LB/FT^3, $\dfrac{kg}{m^3}$)

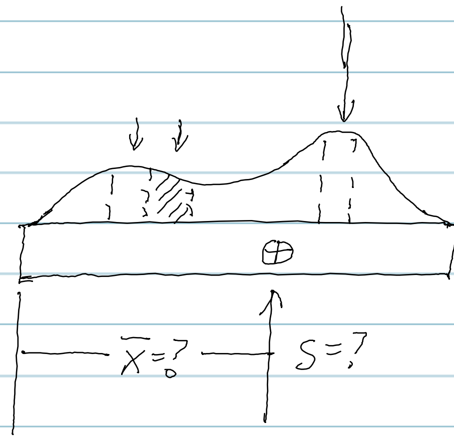
3) AREA PLATE ($\dfrac{LB}{FT^2}$ or $\dfrac{kg}{m^2}$) UNIFORM THICKNESS

4) LENGTH ROD or a BAR ($\dfrac{LB}{FT}$, $\dfrac{kg}{m}$)

SECTION 7.1 (CONT.)

$$\bar{X} = \frac{\sum_{i=1}^{n} x_i F_i}{\sum_{i=1}^{n} F_i}$$

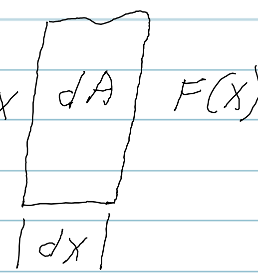
$$A \cong F$$
$$A = KF$$
$$F = \frac{1}{K} A$$

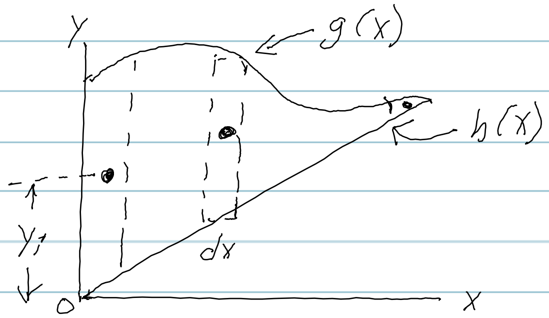


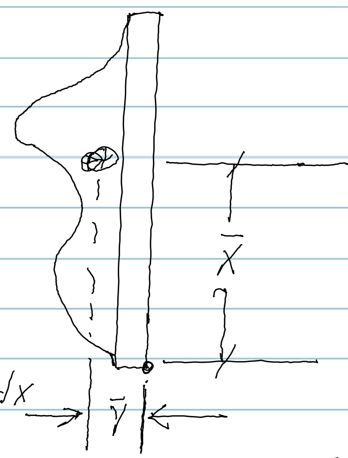
$$\bar{X} = \frac{\sum x_i \left(\frac{1}{K} A_i\right)}{\sum \frac{1}{K} A_i}$$

$$\bar{X} = \frac{\frac{1}{K} \sum x_i A_i}{\frac{1}{K} \sum A_i} = \frac{\int_{x=0}^{x} x_i \left[F(x)\,dx\right]}{\int_{x=0}^{x} F(x)\,dx}$$

$$dA = F(x)\,dx$$







$$\underline{\underline{F(x) = g(x) - h(x)}}$$

$$dA = F(x)\,dx = \left[g(x) - h(x)\right] dx$$

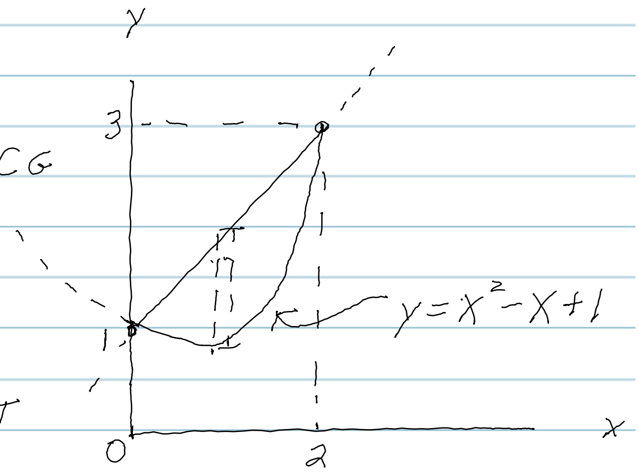
$$\bar{Y} = \frac{\sum y_i A_i}{\sum A_i} = \frac{\int y_i \, F(x)\,dx}{\sum F(x)\,dx} = \frac{\int \left[\frac{1}{2} F(x) + h(x)\right] F(x)\,dx}{\int F(x)\,dX}$$

ANY SITUATION WHERE YOU KNOW BOTH UPPER + LOWER BOUNDRY EQUATION.

$\Rightarrow g(x) + h(x) \Rightarrow h(x) = 0$

SECTION 7.1 PROB. 7-16 CENTROID - CENTER OF MASS

GIVEN: SKETCH



FIND: $\bar{X} + \bar{Y}$ CENTROID, CM, or CG

SOLUTION:

1) FIND UPPER BOUNDRY

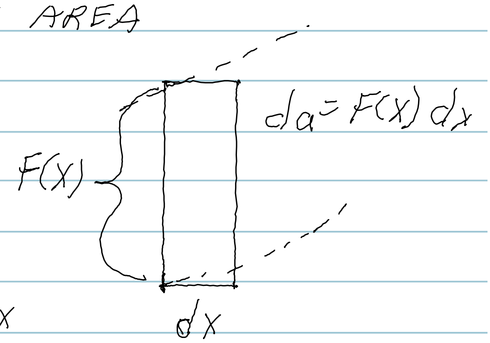
$y = mx + b$ $\qquad$ $b \Rightarrow$ Y INTERCEPT

$b = +1$

$\underline{\underline{y = x + 1}}$ $\qquad$ $m = \text{SLOPE} = \frac{\text{RISE}}{\text{RUN}} = \frac{2}{2} = 1$

2) DRAW DIFFERENTIAL ELEMENT OF AREA

$$F(x) = (x+1) - (x^2 - x + 1)$$
$$F(x) = x + 1 - x^2 + x - 1$$
$$F(x) = -x^2 + 2x$$



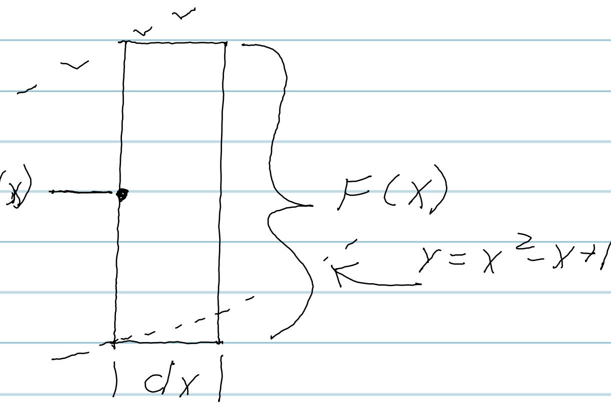
3) $$\bar{x} = \frac{\int x\,F(x)\,dx}{\int F(x)\,dx} = \frac{\int_0^2 x[-x^2+2x]\,dx}{\int_0^2 (-x^2+2x)\,dx}$$

$$\bar{x} = \frac{\int_0^2 (-x^3+2x^2)\,dx}{\int_0^2 (-x^2+2x)\,dx} = \frac{\left.\frac{-x^4}{4} + \frac{2}{3}x^3\right|_0^2}{\left.-\frac{1}{3}x^3 + x^2\right|_0^2}$$

$$\bar{x} = \frac{-\frac{1}{4}2^4 + \frac{2}{3}2^3}{-\frac{1}{3}2^3 + 2^2} = \frac{-4 + 5\tfrac{1}{3}}{-2.67 + 4} = \underline{\underline{1}}$$

NEXT SOLVE FOR "$\bar{Y}$"

SECTION 7.1 PROB 7-16 (CONT.)



1) STEP 1 – FIND $H(x)$

$$H(x) = (x^2 - x + 1) + \tfrac{1}{2} F(x)$$

$$H(x) = x^2 - x + 1 + \tfrac{1}{2}(-x^2 + 2x)$$

$$H(x) = x^2 - x + 1 - \tfrac{1}{2}x^2 + x$$

$$H(x) = \tfrac{1}{2}x^2 + 1$$ (HEIGHT OF CTR $\bar{Y}_{da}$)

NO LONGER "x"

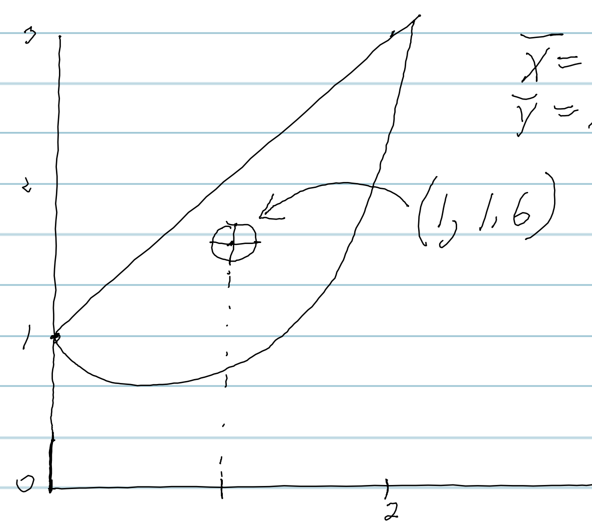
2) STEP – $$\bar{y} = \frac{\int H(x)\,da}{\int da} = \frac{\int H(x)\,F(x)\,dx}{\int F(x)\,dx}$$

$$\bar{y} = \frac{\int_0^2 \left(\tfrac{1}{2}x^2 + 1\right)(-x^2 + 2x)\,dx}{\int_0^2 (-x^2 + 2x)\,dx} = \frac{\int_0^2 \left(-\tfrac{1}{2}x^4 + x^3 - x^2 + 2x\right)dx}{1.333}$$

DONE

$$\bar{y} = \left.\frac{-\tfrac{1}{10}x^5 + \tfrac{1}{4}x^4 - \tfrac{1}{3}x^3 + x^2}{1.333}\right|_0^2 = \frac{-\tfrac{1}{10}2^5 + \tfrac{1}{4}2^4 - \tfrac{1}{3}2^3 + 2^2}{1.333}$$

$$\underline{\underline{\bar{y} = 1.60}}$$

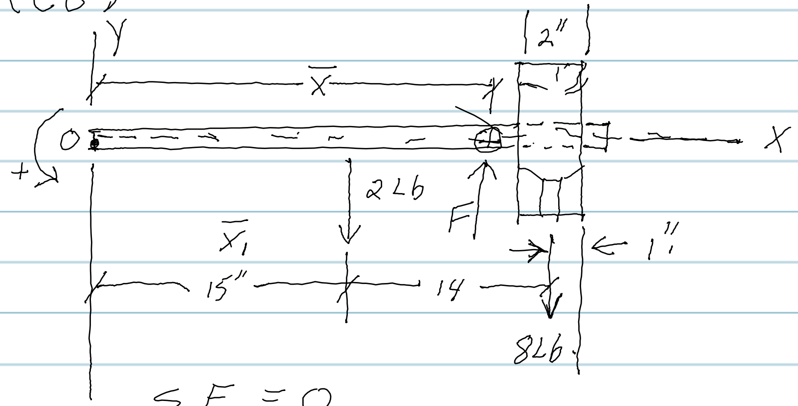


END

SECTION 7.2 COMPOSITE AREA CENTROIDS

& C.M (CG)

INTRO: SYMMETRY
"MIRROR IMAGE"



$\sum_0 M = 0$

$-2(15) + F\bar{x} - 8(29) = 0$

$F\bar{x} = 2(15) + 8(29)$

$\bar{x} = \dfrac{2 \cdot 15 + 8(29)}{F}$

$\sum F_y = 0$

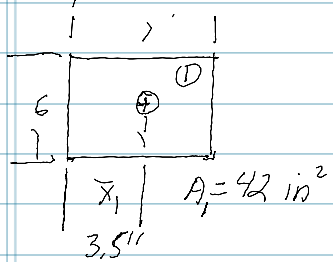
$-2 + F - 8 = 0 \Rightarrow F = 10\,Lb$

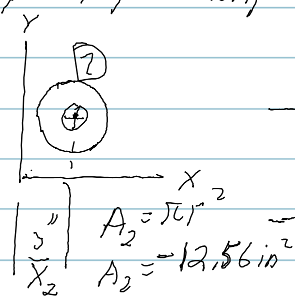
$\bar{x} = \dfrac{2 \cdot 15 + 8(29)}{8+2}$

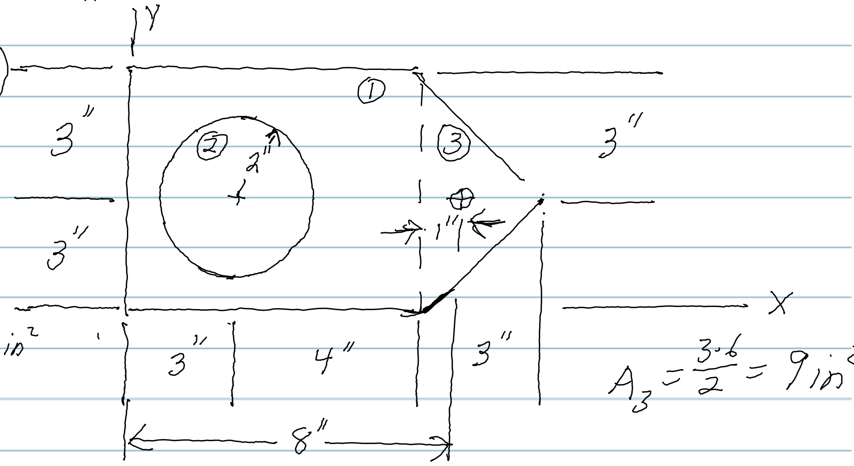
$$\bar{x} = \frac{F_1\bar{x}_1 + F_2(\bar{x}_2)}{F_1 + F_2} = \frac{\sum_{n=1}^{i} F_i\bar{x}_i}{\sum_{n=1}^{i} F_i}$$

SAMPLE PROBLEM:

$\bar{y} = 3$ in (by inspection)

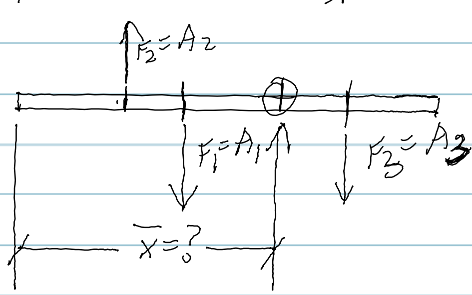






$A_3 = \dfrac{3 \cdot 6}{2} = 9\ in^2$

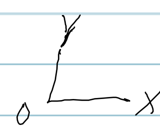
SHAPE	$A(in^2)$	$\bar{x}(in)$	$\bar{x} \cdot A$
1	42	3.5	147
2	-12.56	3.	-37.7
3	9.	8.	72.
$\sum$	$38.43\ in^2$		$181.3\ in^3$



$\bar{x} = \dfrac{\sum \bar{x}A}{\sum A} = \dfrac{181.3\ in^3}{38.43\ in^2}$

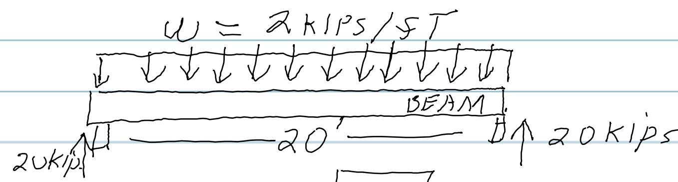
$\bar{x} = 4.72\ in$

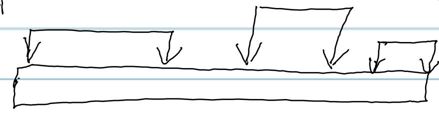
(4.72, 3) in FROM

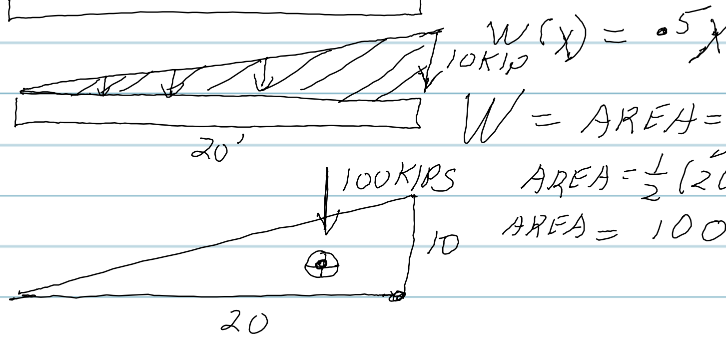


SECTION 7.3 DISTRIBUTED LOADS

POINT LOADS



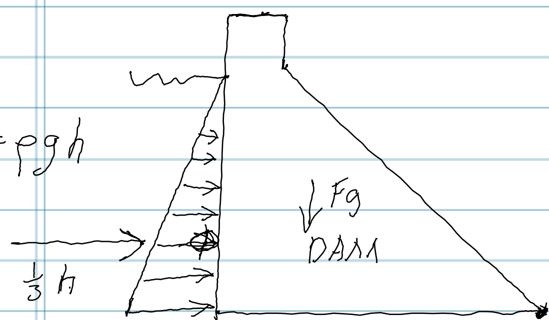




$W = \text{AREA} = \int w(x)\,dx$

$\text{AREA} = \frac{1}{2}(20')\frac{10\,\text{KIP}}{\text{FT}}$

$\text{AREA} = 100\ \text{KIP}$

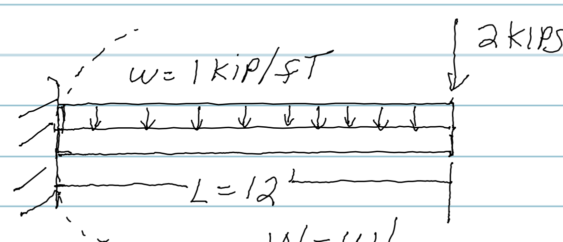


EXAMPLE PROB.

SOLUTION:

1) DRAW FBD

1a) CONVERT ANY DISTRIBUTED LOADS TO POINT LOADS



2) SOLVE FOR REACTIONS

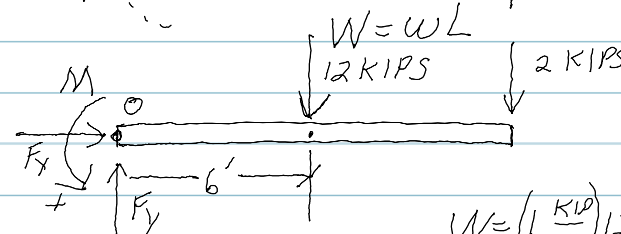
$\Sigma M_0 = 0 \Rightarrow \Sigma F \cdot r_\perp = 0$

$-(12\text{ KIPS})6' - (2\text{ KIPS})12' + M = 0$

$\Sigma M + F \cdot r_\perp = 0$

$-72 - 24 + M = 0$

$M = 96\ \text{KIP-FT}$



$W = \left(1\frac{\text{KIP}}{\text{FT}}\right)12'$

$W = 12\ \text{KIPS}$

$\Sigma F_y = 0 \Rightarrow$

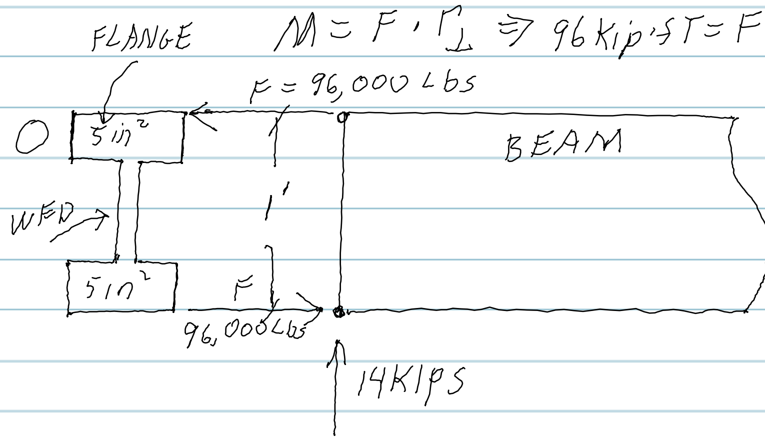
$F_y - 12\text{ kips} - 2\text{ kips} = 0$

$F_y = 14\ \text{KIPS}$

$\Sigma F_x = 0$

$F_x = 0$

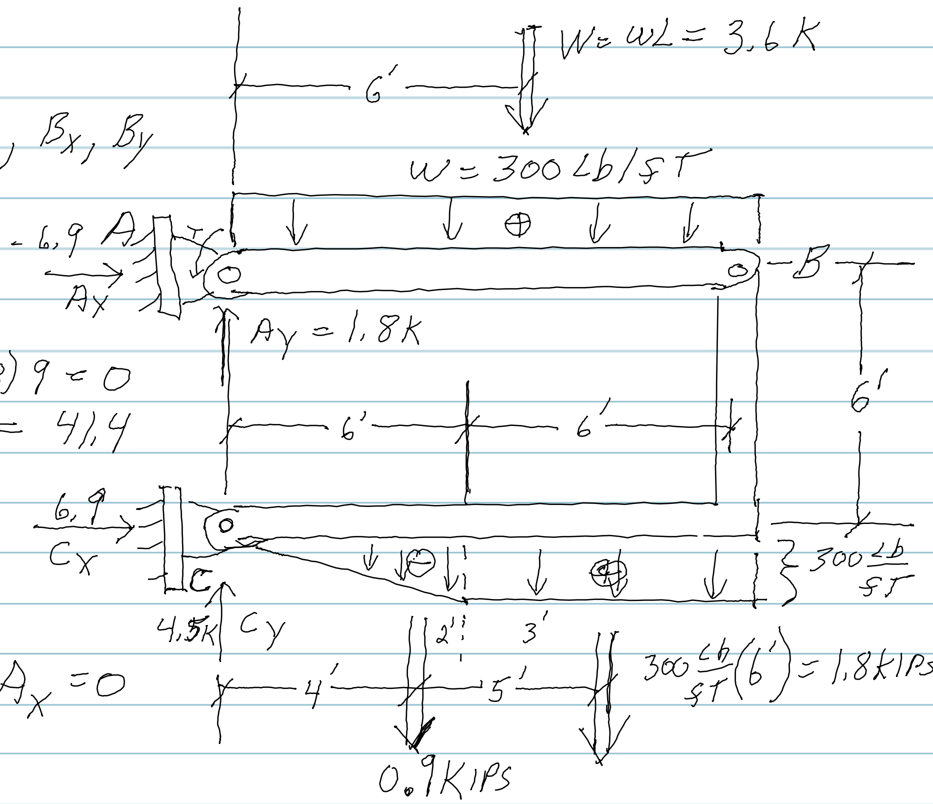
$M = F \cdot r_\perp \Rightarrow 96\ \text{kip·FT} = F \cdot 1\text{FT}$



SECTION 7.3 PROB. 7-55 DISTRIBUTED LOAD ON A FRAME

GIVEN:

FIND: $A_x, A_y, C_x, C_y, B_x, B_y$



SOLUTION:

$\Sigma_A M = 0$

$-(C_x)6' - (.9)4 \quad (3.6)6 - (1.8)9 = 0$

$6C_x = 3.6 + 21.6 - 16.2 = 41.4$

$C_x = \frac{41.4}{6} = 6.9 \text{ KIPS}$

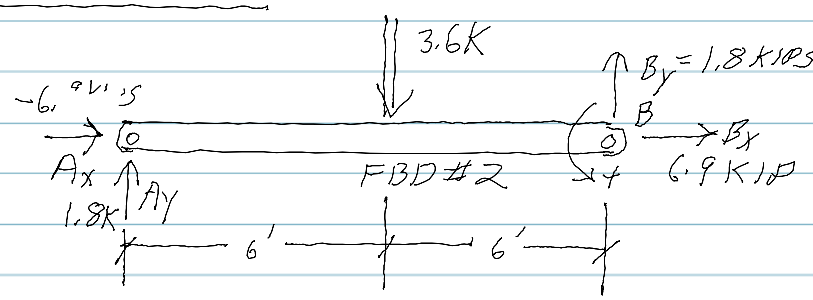
$\Sigma F_x = 0 \Rightarrow (6.9) + A_x = 0$

$A_x = -6.9 \text{ KIPS}$

$\Sigma F_y = 0 \Rightarrow \quad C_y + A_y - .9 - 3.6 - 1.8 = 0$

$C_y + A_y = 6.3 \text{ KIPS}$

FBD # 2



$\Sigma M_B = 0$

$-A_y(12) + 3.6(6) = 0$

$12A_y = 21.6$

$A_y = 1.8 \text{ KIPS}$

$\Sigma F_x = 0$

$+A_x + B_x = 0$

$B_x = -A_x = -(-6.9) = 6.9 \text{ KIPS}$

$\Sigma F_y = 0$

$+1.8 - 3.6 + B_y = 0$

$B_y = 1.8 \text{ KIPS}$

FINALLY:

$C_y + A_y = 6.3 \Rightarrow C_y + 1.8 = 6.3$

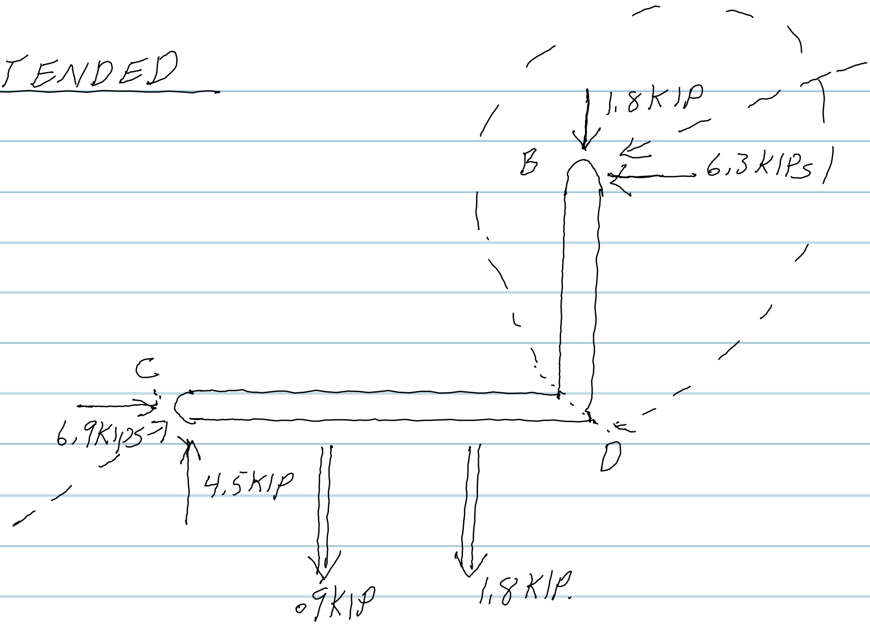
$C_y = 4.5 \text{ KIPS}$

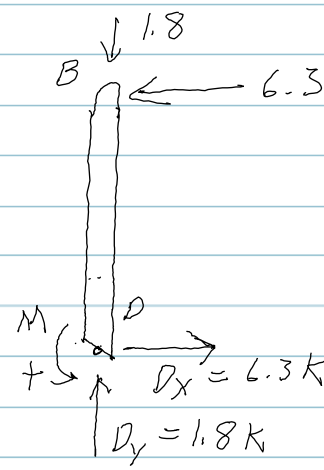
(CONT.)

PROB. 7-55 EXTENDED

WEAKEST LINK

FAILURE MODE

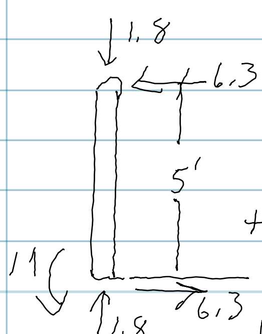




$\Sigma M_D = 0$

$+(6.3)\,6 + M = 0$

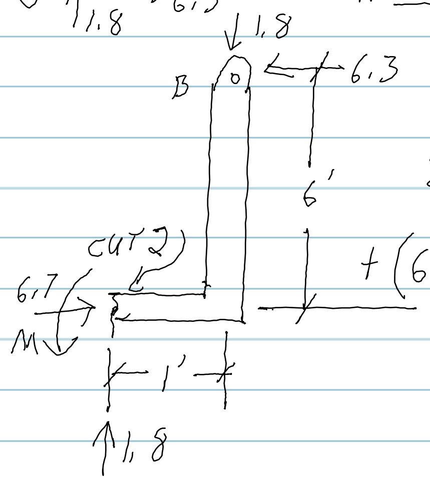
$\underline{\underline{M = -37.8 \text{ kip}\cdot\text{FT}}}$



$\Sigma M_{CUT} = 0$

$+(6.3)5 + M = 0$

$\underline{\underline{M_{CUT} = -31.5 \text{ KIP}\cdot\text{FT}}}$



$\Sigma M_{CUT2} = 0$

$+(6.3)6 - 1.8(1) + M = 0$

$37.8 - 1.8 + M = 0$

$\underline{\underline{M = -36 \text{ KIP}\cdot\text{FT}}}$

SECTION 7.4 CENTROIDS OF VOLUMES + LINES

OPTIONAL SECTION -

$$\bar{x} = \frac{\Sigma\, x\, da}{\Sigma\, da} = \frac{\Sigma\, x\, dV}{\Sigma\, dV}$$

SECTION 7.5 COMPOSITE VOLUMES

$$\bar{x} = \frac{\Sigma (F_i)(\bar{x}_i)}{\Sigma F} = \frac{\Sigma V_i \bar{x}_i}{\Sigma V_i}$$

ASSUME ALL $V \Rightarrow$ CONT. ρ

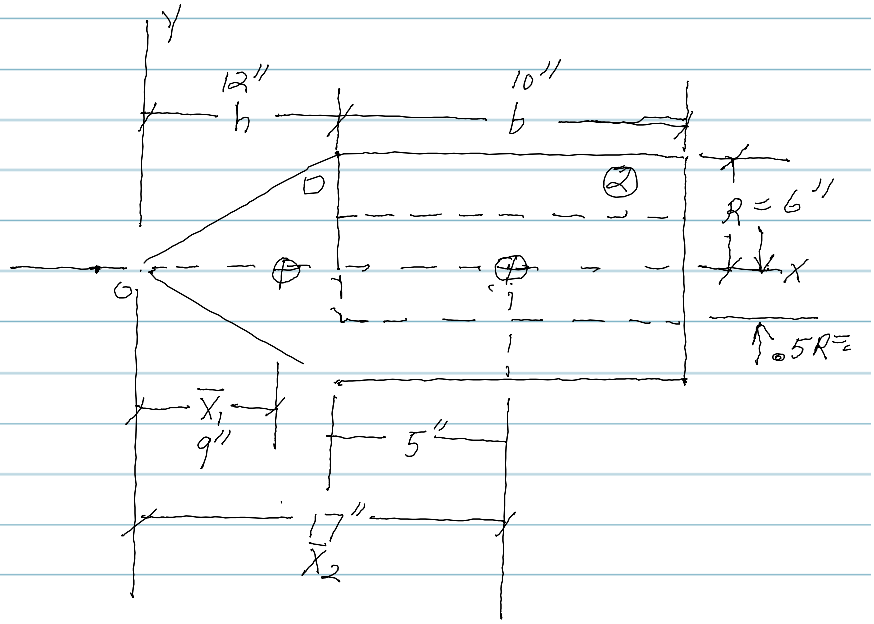
SHAPE	$\bar{x}_i$	V_i	$\bar{x}_i V_i$
1			
2			
3			
Σ		ΣV	$\Sigma \bar{x}_i V_i$

$$\bar{x} = \frac{\Sigma \bar{x}_i V_i}{\Sigma V}$$

SECTION 7.5 PROBLEM 7-71 COMPOSITE VOLUME CENTROID

GIVEN: $h = 12\,in$, $b = 10\,in$
$R = 6\,in$

FIND: $\bar{X}_{CM}$



① CONE

$\bar{x}_1 = \frac{3}{4} h = \frac{3}{4}(12") = 9\,in$

$V_1 = \frac{1}{3}\pi R^2 h = \frac{1}{3}\pi (6)^2 12 = 452\,in^3$

② LARGE CYL,

$\bar{X}_2 = 5" + 12" = 17"$

$V_2 = \pi R^2 b = \pi (6)^2 10 = 1130\,in^3$

③ CYL, HOLE

$\bar{X}_3 = 17\,in$

$V_3 = -\pi (.5R)^2 b = -\pi (3)^2 10 = -283\,in$

SHAPE	$\bar{X}_i$	V_i	$\bar{X}_i V_i$
1	9	452.	4068.
2	17	1130.	19,218.
3	17	-283.	-4811.
Σ		$\Sigma V_i = 1299.$	$\Sigma \bar{X}_i V_i = 18475.$

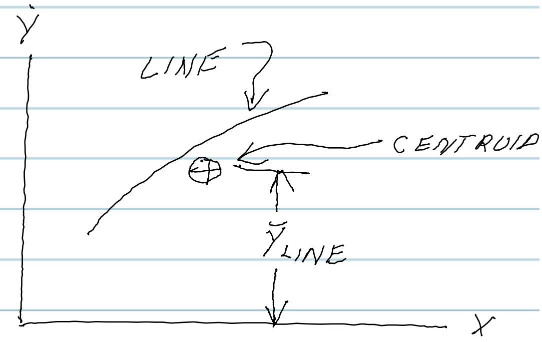
$$\bar{X} = \frac{\Sigma \bar{X}_i V_i}{\Sigma V_i} = \frac{18475.\,in^4}{1299.\,in^3}$$

$$\bar{X} = 14.2\,in$$

END

SECTION 7.6 PAPPUS-GULDINUS THEOREMS

I. AREA OF REVOLUTION (SURFACE AREA)

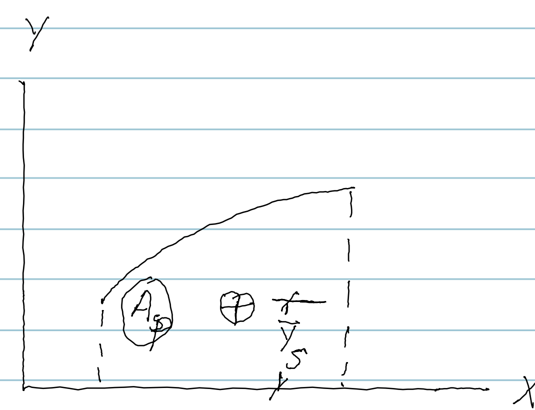


$$* \quad A_S = 2\pi \bar{Y}_L L_{LINE}$$

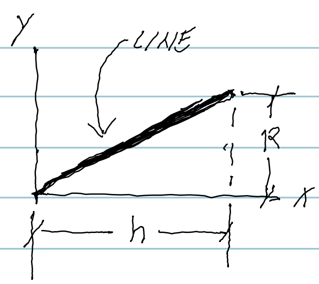
$$\bar{Y} = \frac{\int y\,dL}{\int dL} = \frac{\int y\,dL}{L}$$

$$\bar{y}L = \int f(x)\,dx$$

II $\quad V = 2\pi \bar{Y}_S A_{PLANE\,(X-Y)}$



EXAMPLE:
FIND: $A_S = ?$



SOLUTION: $A_S = 2\pi \bar{Y}_L \cdot L_{LINE}$

$\bar{Y}_L \Rightarrow (\frac{1}{2}h, \frac{1}{2}R) \Rightarrow \bar{Y}_L = \frac{1}{2}R$

$L_{LINE} = \sqrt{h^2 + R^2}$

$$\underline{\underline{A_S = 2\pi\left[\tfrac{1}{2}R\right]\left(\sqrt{h^2+R^2}\right)}}$$

FIND: $V = ?$

SOLUTION: $V = 2\pi \bar{Y}_S A_{PLANE\,(X-Y)}$

$\bar{Y}_S = \frac{1}{3}R \qquad A_{PLANE} = \frac{1}{2}hR$

$$\underline{\underline{V = 2\pi\left(\tfrac{1}{3}R\right)\left(\tfrac{1}{2}hR\right) = \tfrac{2}{6}\pi R^2 h = \tfrac{1}{3}\pi R^2 h}}$$

SECTION 7.7 & 7.8 CENTER OF MASS OF OBJECTS (BY INTEGRATION & BY COMPOSITE PARTS)

GOOD NEWS:

$$\bar{x} = \frac{\sum x_i F_i}{\sum F_i} = \frac{\int x_i \, dF}{\int dF} = \frac{\int x \, dm}{\int dm}$$

DENSITY

$$\rho = \frac{m}{V} \Rightarrow m = \rho \cdot V$$

$\frac{kg}{m^3}, \frac{gm}{cm^3}$,

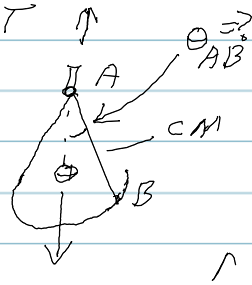
$$dm = d(\rho \cdot V)$$

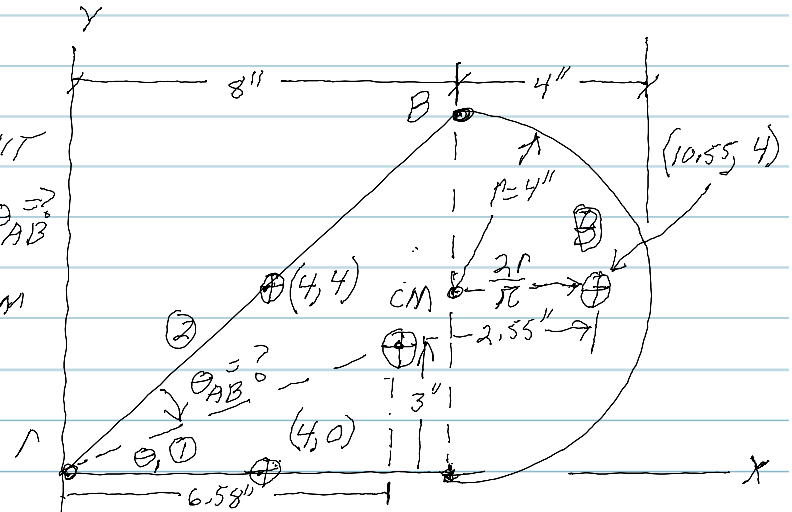
SECTION 7.8 + 7.9 PROB. 7-137 SOLID BAR

GIVEN:

FIND: $\theta_{AB-VERT}$ WHEN POINT A IS SUPPORT ↑

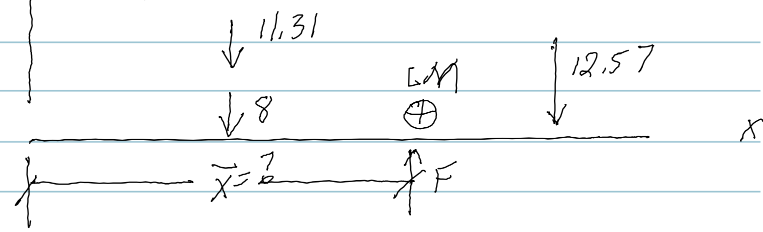
SOLUTION:





$L_1 = 8\,in \qquad L_2 = \sqrt{8^2+8^2} = 11.31\,in$

$L_3 = \frac{1}{2}(2\pi r) = \pi(4) = 12.57\,in$



SHAPE	$\bar{X}_i$	$\bar{Y}_i$	L_i (A)	$\bar{X}_iL_i$	$\bar{Y}_iL_i$
①	4	0	8	32	0
②	4	4	11.31	45.24	45.24
③	10.55	4	12.57	132.6	50.28
Σ			31.88	209.8	95.52

$\bar{X}_{CM} = \frac{\Sigma \bar{x}_i L_i}{\Sigma L_i} = \frac{209.8}{31.88} = \underline{\underline{6.58\,in}}$

$\bar{Y} = \frac{95.52}{31.88} = \underline{\underline{3.00\,in}}$

$CM = CG = (6.58, 3)\,in$

FIND: $\theta_1 = ?$ $\quad TAN(\theta_1) = \frac{3}{6.58}$

$\theta_1 = 24.5^\circ$

$\underline{\underline{\theta_2 = 45^\circ - 24.5^\circ = 20.5^\circ}}$

NOTE: GENERAL PRINCIPLE – C.M. WILL BE DIRECTLY BELOW LIFT POINT – IF NO OTHER FORCE ACT