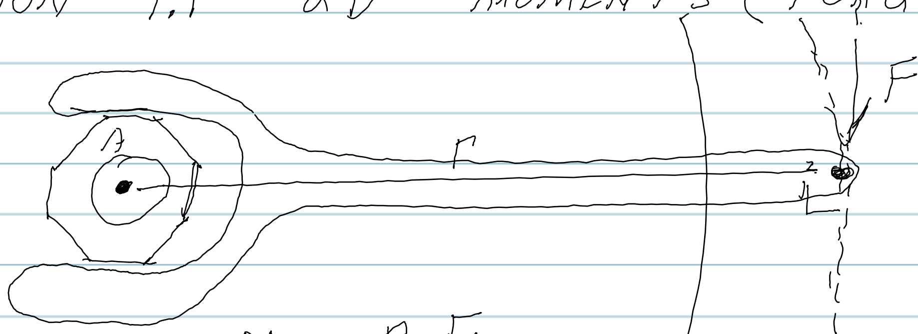
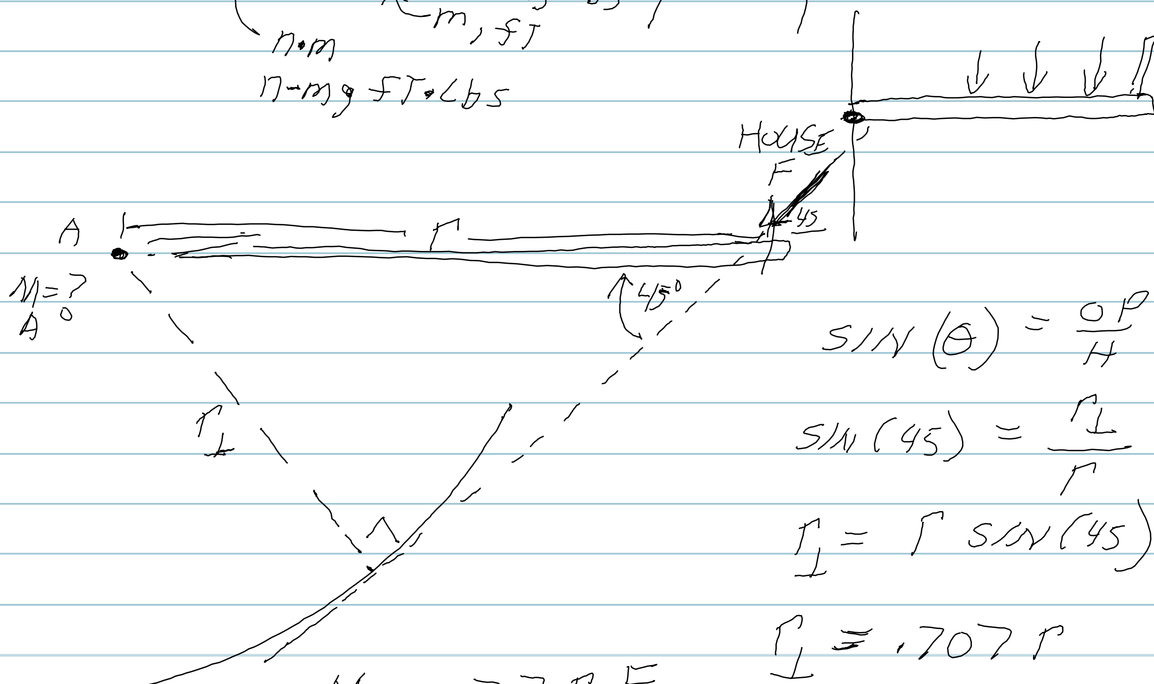
4.4 FORCES AND MOMENTS

SECTION 4.1 2D - MOMENTS (TORQE)



$M = r_{\perp} F$ ← n, lbs ; $r_{\perp}$ ← m, ft ; M ← n·m

n-m, ft·lbs



$\sin(\theta) = \frac{OP}{H}$

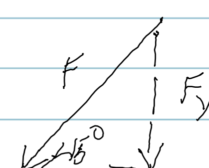
$\sin(45) = \frac{r_{\perp}}{r}$

$r_{\perp} = r \sin(45)$

$r_{\perp} = .707\, r$

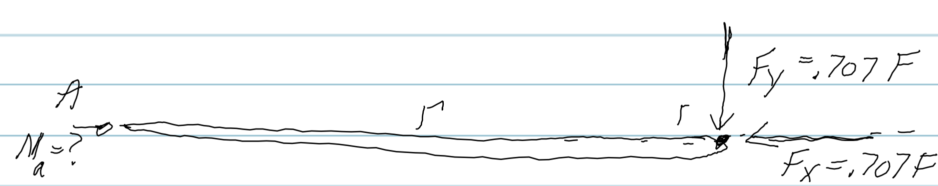
$M_a = .707\, r\, F$

COMPONENTS



$F_x = .707 F$

$F_y = .707 F$



$M = r_{\perp y} F_y + r_{\perp x} F_x = r[.707F] + 0[.707F]$

$\underline{M = .707 F r}$

SECTION 4.1 (CONT)

DIRECTION OF A MOMENT

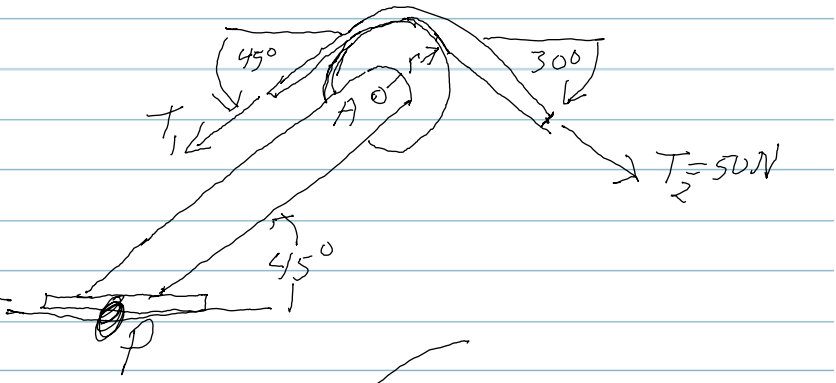
2D –

CW – TIGHTEN

PROB. 4-89

GIVEN: $|r_{AP}| = 650\ mm$

$r = 120\ mm$

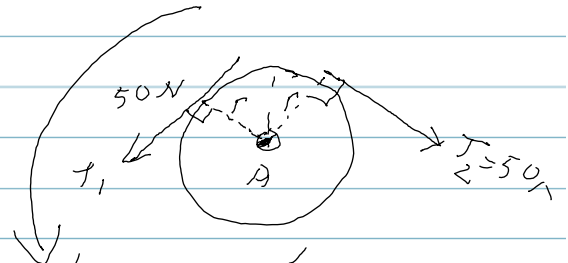


FIND: $M_A = ?$, M_P

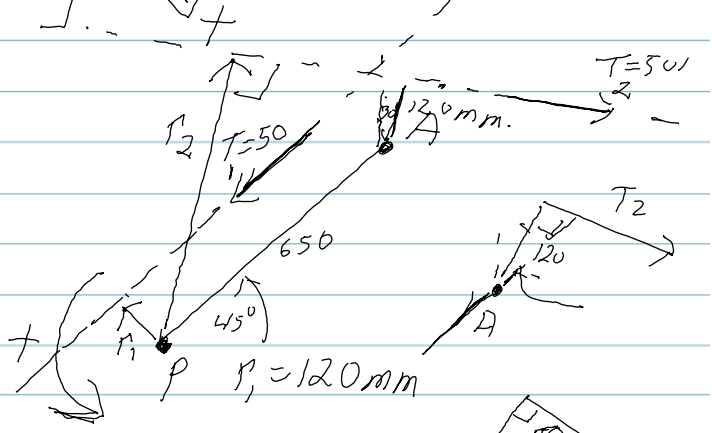
a) $M_a = r_1 T_1 + r_2 T_2$

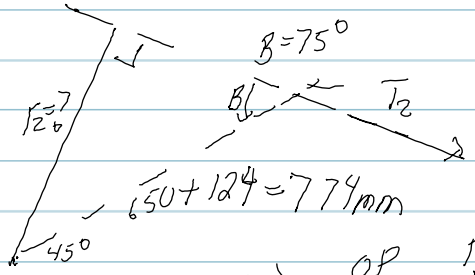
$M_a = +(120mm)\ 50N - [+(120mm)\ 50N]$

$M_a = 0$



b) $\Rightarrow M_P$





$650 + 124 = 774mm$

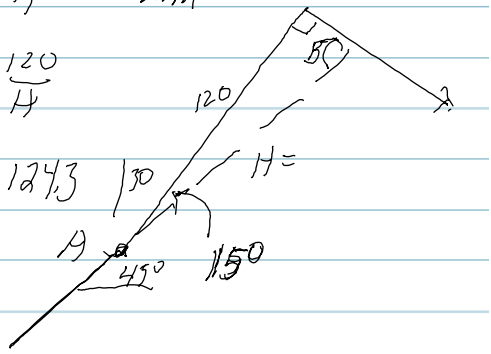
$\sin(B) = \frac{OP}{H} = \frac{r_2}{774}$

$r_2 = 774 \sin(75°)$

$r_2 = 747.6 = 748.\ mm$

$\cos(15) = \frac{120}{H}$

$H = \frac{120}{.9659} = 124.3$



PROB. 4-9 (CONT.)

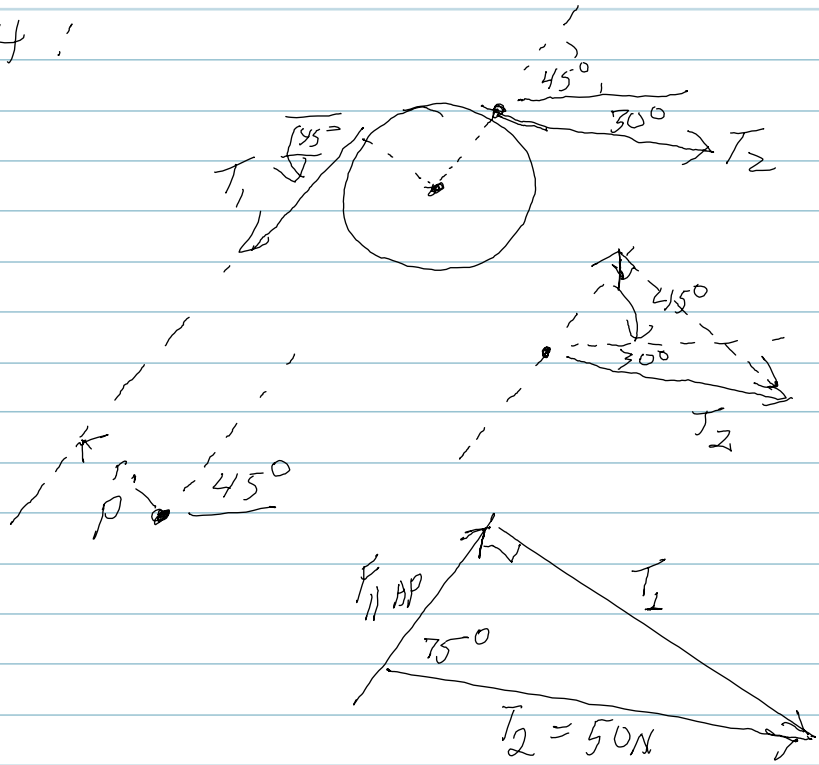
$M_p = r_1 T_1 + r_2 T_2 = +(120)\,50 - (748)(50)$

$M_p = +6000 - 37{,}400 = -25{,}400.\ N\cdot mm$ ← ERROR

$31{,}400\ N\cdot mm = 31.4\ N\cdot m$

$M_p = -25.4\ N\cdot m$

EASY APPROACH:



$\sin(75) = \frac{T_1}{50}$

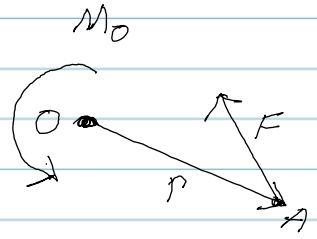
$T_1 = 50\sin(75) = 48.29\ N$

$M_p = r_1 T_1 + 0(F_{\parallel AP}) - r T_1 = 120(50) - 774(48.29\ N)$

$M_p = +6000 - 37391 = 31{,}391\ N\cdot mm = 31.4\ N\cdot m$

CH. 4 SECTION 4.~~2~~ 3D MOMENT VECTOR

$M_O = r_{OA} \times F$



CROSS PRODUCT – (SECTION 2.5, pg 10)

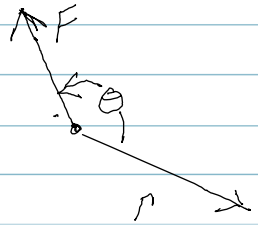
DIRECTION IN 3D

M_O IS $\perp$ PLANE $r \times F$

$M_O = A\,i + B\,j + C\,k \qquad |M_O| = \sqrt{A^2 + B^2 + C^2}$

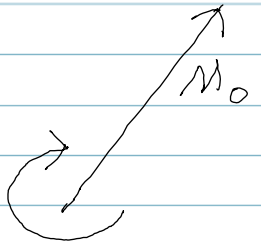
$e_M = \dfrac{M_O}{|M_O|}$

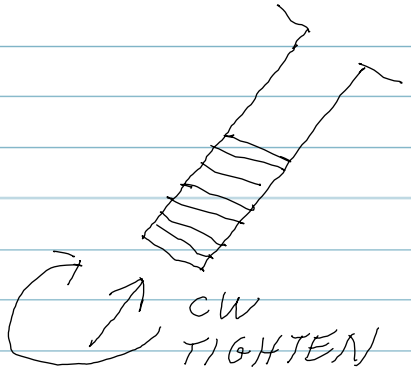
MOMENT VECTOR

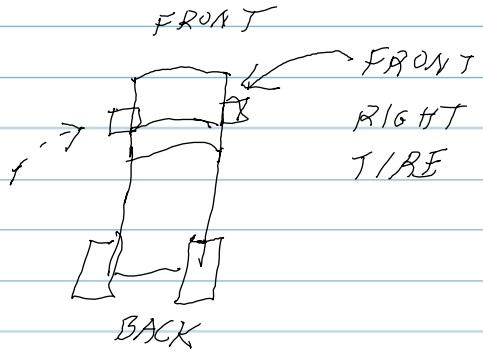


RIGHT HAND
1) FINGER — r DIRECTION
2) FLEX FINGER θ

RHR







POSITION – BACK

$|M| = |r_\perp| \cdot |F|$

$|r_\perp| = \dfrac{|M|}{|F|} \Leftarrow$ 3D

$M = r \times F = |r||F| \sin(\theta)$

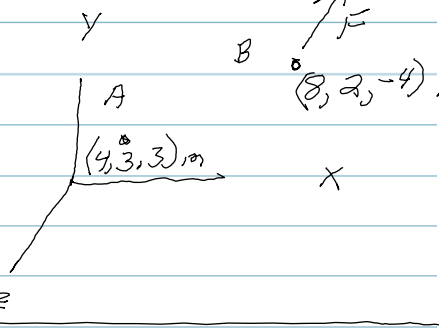
SECTION 4.2 (CONT)

VARIGON'S THEOREM

$$M = r \times F = (r \times F_i) + (r \times F_j) + (r \times F_k)$$

PROBLEM 4-59 3D MOMENTS

GIVEN: $F = 30i + 20j - 10k$ (N)



FIND: $M_A = ?$ DUE TO "F", $|M_O|_{MAX} = ?$

SOLUTION:

a) $M_A = r_{AB} \times F$

$$r_{ab} = \{8i + 2j - 4k\} - \{4i + 3j + 3k\}$$

$$r_{ab} = 4i - 1j - 7k$$

$$M_A = \begin{vmatrix} i & j & k \\ 4 & -1 & -7 \\ 30 & 20 & -10 \end{vmatrix} = 150j - 170j + 110k \ (N \cdot m) =$$

$$|M_A| = \sqrt{150^2 + 170^2 + 110^2} = 252 \ N \cdot m$$

b) $M_{max} = |r_{ab}||F| \sin(\theta) \Rightarrow M_{max} = |r_{ab}||F|$ when $\theta = 90$

$$|r_{ab}| = \sqrt{4^2 + (-1)^2 + (-7)^2} = 8.12 \ m$$

$$|F| = \sqrt{30^2 + 20^2 + (-10^2)} = 37.4 \ N$$

$$M_{max} = (37.4 N)\ 8.12 \ m$$

$$M_{max} = 304 \ (N \cdot m)$$

SECTION 4.3 (CONT.) MOMENT OF A FORCE ABOUT A LINE

INTRO:

1) START WITH FIND $M_{\text{POINT ON LINE}}$ M_P

$M_P = r \times F$ $r \Rightarrow$ ANY POINT ON THE LINE TO ANY POINT ON LINE OF THE VECTOR

2) FIND PROJECTION M_P // TO THE LINE

DOT PRODUCT (NOTE Pg 8 - SEC 2.4)

MAG. $\rightarrow$ $|M_{LINE}| = e_{LINE} \cdot M_P$ GIVES: $|M_{LINE}|$ or TORQUE TEND TO TWIST

VIP: $+|M_{LINE}| \Rightarrow M_{//}$ POINT IN THE DIRECTION OF e_{LINE} - USE RHR

3) YOU WANT THE COMPONENTS OF M_{LINE}
USE UNIT VECTOR $\Rightarrow$ $e = \frac{F}{|F|}$

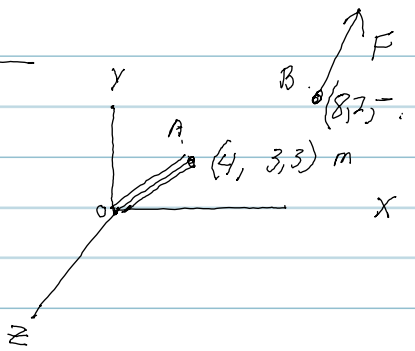
$M_{LINE} = e_{LINE}(|M_{LINE}|)$

M_{LINE} ← VECTOR; $e_{LINE}(|M_{LINE}|)$ ← MULT.

4) SHORTCUT: $M_{LINE} = e_{LINE}[e_{LINE} \cdot (r \times F)]$

$(r \times F)$: MIXED TRIPLE PRODUCT (NOTES - Pg 10, SEC. 2.5)

SECTION 4.3 EXAMPLE PROBLEM



EXTEND PROB 4-59 (SEE NOTES Pg. 20)

TO FIND: $|M_{LINE}|_{OA} = ?$

GIVEN: $F = 30i + 20j - 10k$ (N)

$r_{ab} = 4i - 1j - 7k$ (m)

$M_p = M_a = 150i - 170j + 110k$ (N·m)

SOLUTION:

$$|M_{LINE}|_{OA} = e_{LINE_{OA}} \cdot M_p$$

$$r_{OA_{LINE}} = 4i + 3j + 3k \text{ (m)}$$

$$|r_{OA_{LINE}}| = \sqrt{4^2+3^2+3^2} = 5.831$$

$$e_{OA_{LINE}} = \frac{r_{OA}}{|r_{OA}|} = \frac{4i+3j+3k}{5.831}$$

$$e_{OA} = .6860i + .5145j + .5145k$$

$$|M_{LINE}| = \begin{matrix} .6860i & +.5145j & +.5145k \\ \cdot\ 150i & -170j & +110k \end{matrix}$$

$$|M_{LINE}| = .686(150) - 170(.5145) + 110(.5145)$$

$$|M_{LINE}| = +72.04 \text{ (N·m)}$$

b) $$|M_{LINE}| = e_{LINE} \cdot (r_{AB} \times F) = \begin{vmatrix} e_x & e_y & e_z \\ r_{abx} & r_{aby} & r_{abz} \\ F_x & F_y & F_z \end{vmatrix}$$

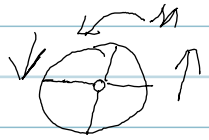
$$|M_{LINE}| = \begin{vmatrix} .686 & .5145 & .5145 \\ 4 & -1 & -7 \\ 30 & 20 & -10 \end{vmatrix}$$

$$|M_{LINE}| = 72.03 \text{ (N·m)}$$

$$M_{LINE} = e_{LINE}\left(|M_{LINE}|\right) = 72.03(.686i + .5145j + .5145k)$$

SECTION 4.4 COUPLES

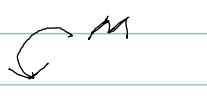
DEF: 2 FORCES
a) SAME MAG.
b) OPPOSITE DIRECTION
c) SOME SEP.

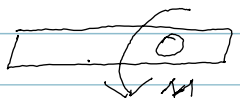


PROPERTIES:

$M_{\text{COUPLE}} = r \times F$ — DISP VECTOR FROM THE LINE OF ACTION OF A "-F" TO ANY POINT ON THE SECOND "+F" LINE OF ACTION *

VIP: $M_C \Rightarrow$ INDEPENDENT OF ITS LOCATION



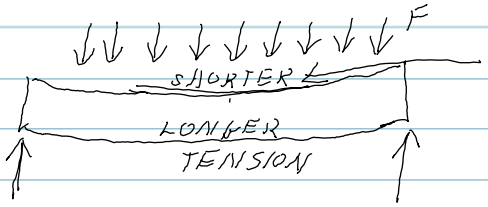


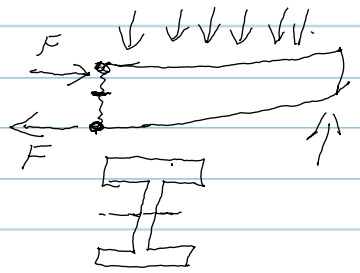
$M = r_1(F)$

2 D $\Rightarrow$

$M_0 = r_1 F = -A_1 F_1 - A F_2$
$|F_1| = |F_2|$
$M_0 = -2AF$
$M_0 = -r_1 F = -d(F)$

BEAM

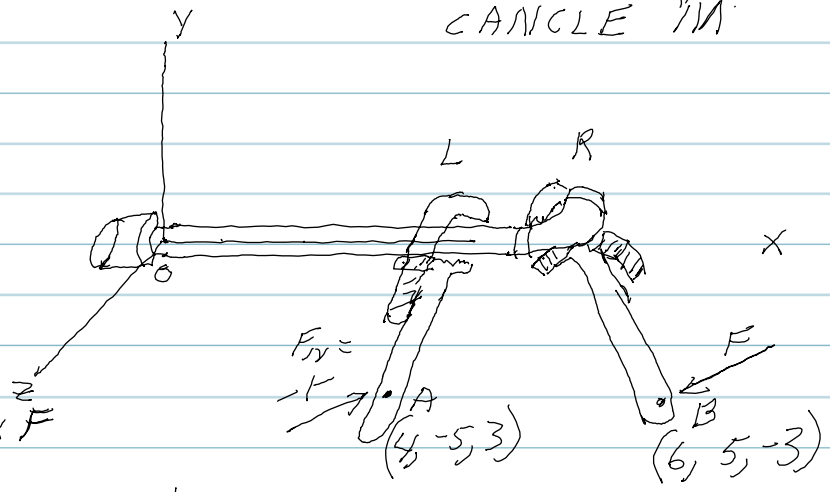




* VIP $\Rightarrow$ r GOES FROM -F TO +F, YOU CHOSE EITHER F TO BE (+)

SECTION 4.4 (CONT) PROBLEM 4-127 COUPLE ↺ CANCLE "M"

GIVEN: $F = 10\ K\ (Lb)$



FIND: $\Sigma M_O = ?$

SOLUTION:

$$\Sigma M_O = r_{OA} \times (+F_N) + r_{OB} \times F$$

$$M_{OL} = r_{OA} \times F_N = \begin{vmatrix} i & j & K \\ 4 & -5 & 3 \\ 0 & 0 & -10 \end{vmatrix} \begin{matrix} i & j \\ 4 & -5 \\ 0 & 0 \end{matrix} = +50i - [-40j]$$

$$M_{OL} = \underline{50i + 40j}\ (in \cdot Lbs)$$

$$M_{OR} = r_{OB} \times F = \begin{vmatrix} i & j & K \\ 6 & -5 & -3 \\ 0 & 0 & +10 \end{vmatrix} \begin{matrix} i & j \\ 6 & 5 \\ 0 & 0 \end{matrix} = -50i - 60j\ (in \cdot Lbs$$

$$\Sigma M_O = M_{OL} + M_{OR} = (+50i + 40j) + (-50i - 60j) = \underline{-20j\ in}$$

FURTHER REDUCTION IN M_O = POSSIBLE ??

LEFT WRENCH ⇒ MOVE FROM $A = (4, -5, 3)$
TO $A = (5, -5, 3)$

$$M_{OL} = r_{OA} \times F_N = \begin{vmatrix} i & j & K \\ 5 & -5 & 3 \\ 0 & 0 & -10 \end{vmatrix} \begin{matrix} i & j \\ 5 & -5 \\ 0 & 0 \end{matrix} = +50i - (-50j) = \underline{+50i + 50j}$$

NEW

$$\Sigma M_O = (-50i - 60j) + (50i + 50j) = \underline{-10j\ (in \cdot Lbs)}$$

SECTION 4.4 (CONT.) PROBLEM 4-127 (CONT.)

FIND: $M_C = ?_O = r \times F = r_{AB} \times (-F_{AB})$

$r_{ab} = (6i - 5j - 3k) - (4i - 5j + 3k)$

$r_{ab} = +2i + 0j - 6k$

$$\begin{vmatrix} i & j & k \\ +2 & 0 & -6 \\ 0 & 0 & 10 \end{vmatrix} \begin{matrix} i & j \\ +2 & 0 \\ 0 & 0 \end{matrix} = -[20j]$$

$= -20j$ (in·lbs)