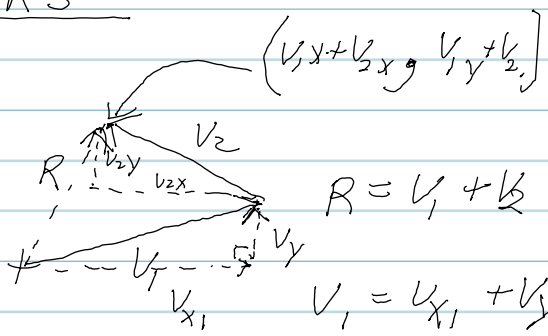
CH. 2 SCALARS & VECTORS

2.1 u |u|

ADDITION - GRAPHICAL



$R = V_1 + V_2$

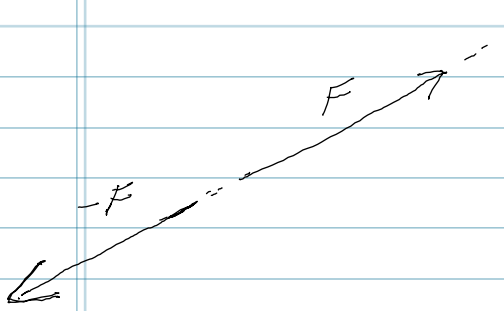
$V_1 = V_{x1} + V_{y1}$

$V_1 = V_{1x}\,i + V_{1y}\,j$

$V_2 = V_{2x}\,i + V_{2y}\,j$

$R = (V_{1x}+V_{2x})\,i + (V_{1y}$

∴ COMPONENT:



PRODUCT: SCALAR & VECTOR

$$a(V) = a(V_1\hat{i} + V_2\hat{j} + V_3\hat{k}) =$$

$$a(V) = aV_1\,i + aV_2\,j + aV_3\,k$$

UNIT VECTOR $\quad u = |u|\,e$ ← NUMBER (+), ← UNIT VECTOR

$u = 1\,i + 2\,j + 3\,k \qquad e = \dfrac{u}{|u|} =$

$a^2 + b^2 = c^2$

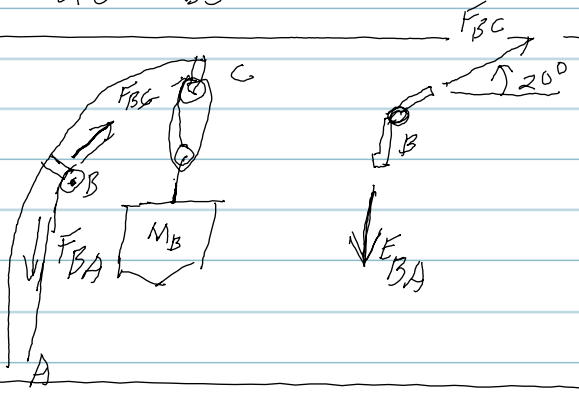
$$|u|^2 = (u_x)^2 + (u_y)^2 + (u_z)^2$$

$$|u| = \sqrt{u_x^2 + u_y^2 + u_z^2}$$

CH 2.1 PROBLEM

2-12 GIVEN: $F_{BA} + F_{BC} = 200 \angle \text{lbs}$

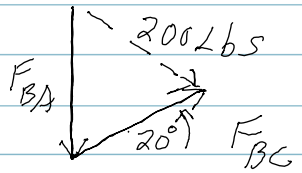
SKETCH:



FIND: $|F_{BA}| = ?$ USE GRAPHICAL METHOD

SOLUTION:

$$|F_{BA}| = |F_{BC}|$$



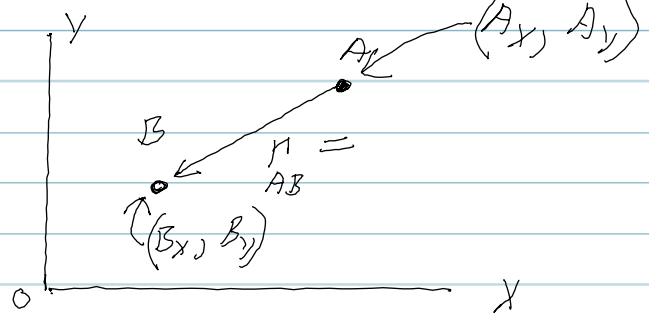
$F_{BA} = 170 \angle \text{lbs}$

SECTION 2.2 COMPONENTS IN TWO DIMENSIONS

$$u = u_x + u_y = u_x i + u_y j$$

FORTRAN

$$|u| = \sqrt{u_x^2 + u_y^2}$$



$$r_{AB} = (B_x - A_x) i + (B_y - A_y) j$$

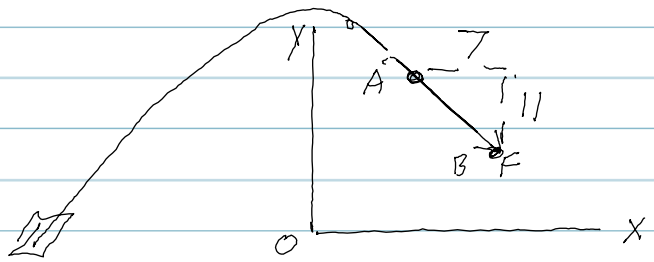
$$r_{AB} = -r_{BA}$$

SECTION 2.2 PROBLEM

2-23 PROB.

GIVEN: $|F| = 10 \text{ Lbs}$

SKETCH:



FIND: $F = \_\_\_ i \_\_\_ j$

SOLUTION: $F = |F| e \qquad e = ?$

$$r_{AB} = 7i - 11j \qquad r_{AB} = |r_{AB}| e_{AB}$$

$$|r_{AB}| = \sqrt{7^2 + (-11)^2} = 13.04$$

$$e_{AB} = \frac{r_{AB}}{|R_{AB}|} = \frac{7i - 11j}{13.04} = 0.5368\, i - 0.8436\, j$$

$$e = e_{AB} \Rightarrow F = |F| e$$

$$F = (10 \text{ Lbs})(0.5368\, i - 0.8436\, j) = \underline{5.37\, i - 8.44\, j \text{ Lbs}}$$

SECTION 2.3 COMPONENTS IN 3 DIMENSIONS

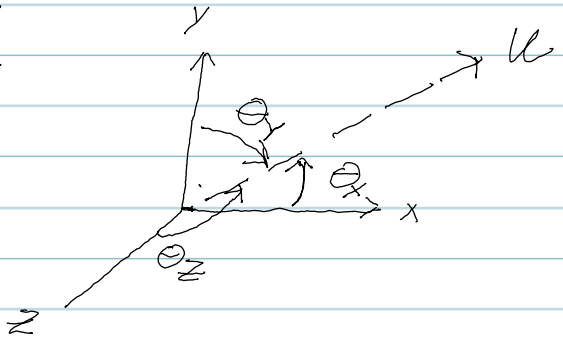
(1) $|U| = \sqrt{U_x^2 + U_y^2 + (U_z)^2}$ (2) $F = |F| e$

$e_F = F = 0.5368\,i - .8436\,j$

$e_F = F = \cos(\theta_x)\,i - \cos(\theta_y)\,j$

$e_F = F_x = .5368 = |U| \cos(\theta_x)$

$e_F = 1 \cos(\theta_x)$



(3) $e = \cos(\theta_x)\,i + \cos(\theta_y)\,j + \cos(\theta_z)\,j$

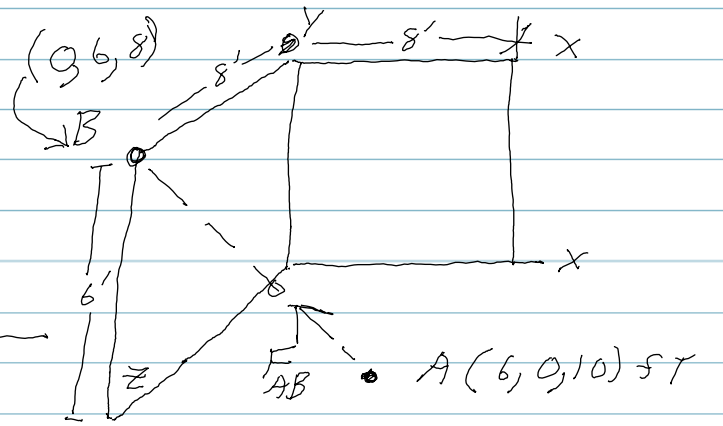
$r_{AB} = (X_B - X_A)\,i + (Y_B - Y_A)\,j + (Z_B - Z_A)\,K$

PARALLEL LINES IN 3D $e_1 = e_2$

2-91 PROBLEM

GIVEN: $|F_{AB}| = 200$ Lbs

FIND: $\bar{F}_{AB} = ?$



$\bar{F}_{AB} = |F_{AB}|\, e_{AB} \Rightarrow e_{AB} = ?$

$r_B = 0\,i + 6\,j + 8\,k$

$-r_a = +6\,i + 0\,j + 10\,k$

$r_{ab} = -6\,i + 6\,j - 2\,k$

$|r_{ab}| = \sqrt{6^2 + 6^2 + 2^2} = 8.718$

$e_{AB} = \dfrac{\bar{r}_{ab}}{|r_{ab}|} = \dfrac{-6\,i + 6\,j - 2\,k}{8.718}$

$e_{AB} = -.6882\,i + .6882\,j - .2294\,k$

$F_{AB} = 200 \text{ Lbs}\,(-.6882\,i + .6882\,j - .2294\,k)$

$F_{AB} = -138\,i + 138\,j - 45.9\,K$ Lbs

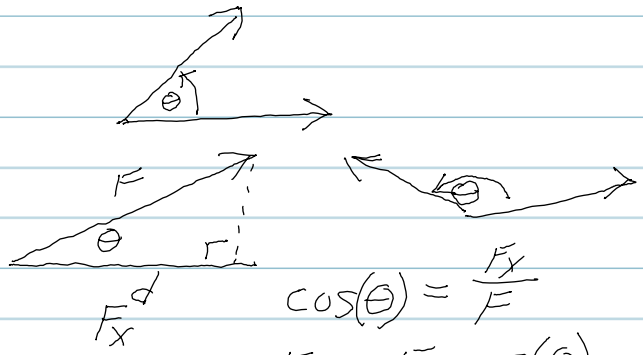
2.4 DOT PRODUCT

$$u \cdot v = |u||v| \cos(\theta)$$

$$W = |F| \cdot |d| \cos(\theta)$$

(CONSTANT)

$$W_{AB} = \bar{F} \cdot \bar{r}_{AB}$$



$$\cos(\theta) = \frac{F_x}{F}$$

$$F_x = F \cos(\theta)$$

CASE: $\perp \Rightarrow \theta = 90 \Rightarrow \cos(90) = 0$

SPECIAL CASE $u \cdot v = 0$ when $u \perp v$

CASE: $u \parallel v \Rightarrow \theta = 0 \Rightarrow \cos(0) = 1$

$u \cdot v = |u||v|$ when $u \parallel v$

EXAMPLE: IF $\perp \Rightarrow u \cdot v = 0$, ALT. $|u||v| \cos(\theta)$

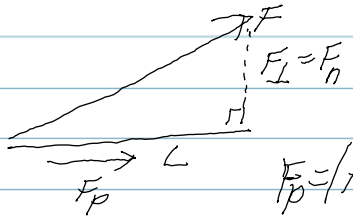
KEY: $u \cdot v = u_x v_x + u_y v_y + u_z v_z = \_\_\_\_$

VECTOR COMPONENTS $\parallel$ & $\perp$ TO A LINE

$$F_p = F \cdot e_L = |F||e_L| \cos(\theta)$$

$$F_p = |F| \cos(\theta) \Rightarrow * F_p = F \cdot e_L$$

MAGNITUDE OF $\parallel$ F TO LINE



$F_p = |F| \cos(\theta)$

$$* \bar{F}_p = (F \cdot e_L) e_L$$

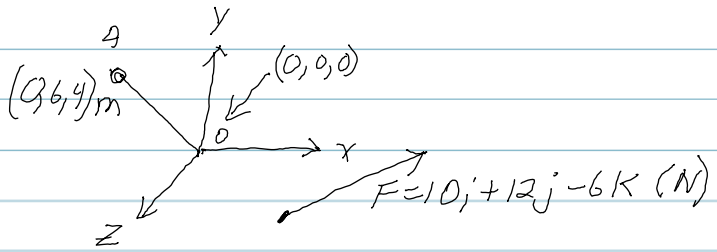
VECTOR FORCE $\parallel$ TO LINE

$$\bar{F}_p + \bar{F}_N = \bar{F}$$

$$* F_N = \bar{F} - \bar{F}_p$$

PROBLEM #2-116

GIVEN:



FIND: $\bar{F}_{\parallel_{OA}} = ?$, $\bar{F}_{n_{aa}} = ?$

SOLUTION: PLAN $\bar{F}_{\parallel_{OA}} = (\bar{F} \cdot e_{LOA})\, e_{LOA}$

$$r_{oa} = \begin{cases} r_a = 0i + 6j + 4k \\ -r_o = 0\ \ 0\ \ 0 \\ r_{oa} = 0i + 6j + 4k \end{cases}$$

$$|r_{oa}| = \sqrt{6^2+4^2} = 7.21$$

$$e_{oa} = \frac{r_{oa}}{|r_{oa}|} = .832j + .555k$$

VECTOR FORCE "F" // TO LINE "OA"

$$F_{\parallel_{oa}} = (\bar{F} \cdot e_{LOA})\, e_{LOA}$$

$$F_{\parallel_{oa}} = \left[(10i + 12j - 6k) \cdot (.832j + .55k)\right] e_{LOA} = 6.656\, e_{Loa}$$

$$F_{\parallel_{oa}} = (6.656)(.832j + .555k) = \underline{0i + 5.54j + 3.69k}\ (N)$$

FIND: $\bar{F}_N$ TO LINE "OA"

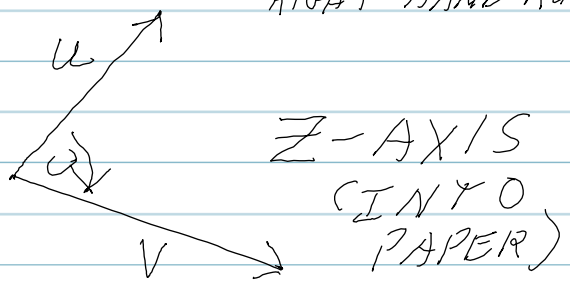
$$F_N = \bar{F} - \bar{F}_p = [10i + 12j - 6k] - [0i + 5.54j + 3.69k]$$

$$F_N = \underline{10i + 6.46j - 9.69k}\ (N)$$

2.5 CROSS PRODUCT

DEFINITION: $u \times v = |u||v| \sin(\theta)\, e_\perp$ ← DEFINED RIGHT HAND RULE

$$u \times v = (u_y v_z - u_z v_y)\, i - (u_x v_z - u_z v_x)\, j + (u_x v_y - u_y v_x)\, k$$



$$u \times v = \begin{vmatrix} i & j & k & i & j \\ u_x & u_y & u_z & u_x & u_y \\ v_x & v_y & v_z & v_x & v_y \end{vmatrix}$$

$u \times v = +[\text{FIRST 3 TERMS}] - [\text{BACK 3 TERMS}]$

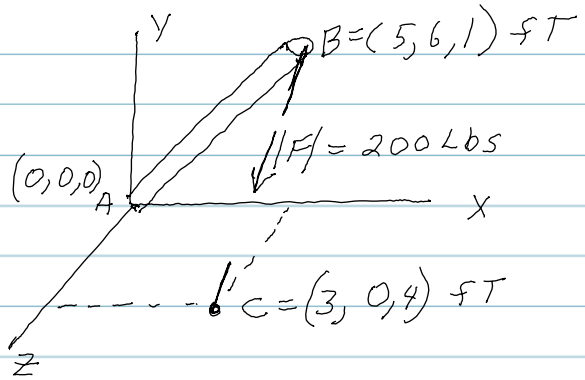
MIXED TRIPLE PRODUCT

$$R = u \cdot (v \times w) = \begin{vmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{vmatrix}$$

PROBLEM 2-158

GIVEN:

FIND: $r_{ab} \times F = ?$, $r_{ac} \times F = ?$



SOLUTION: $F = |F| e_{bc}$

$e_{bc} = ?$

$F = (200 \text{ Lbs})(-.2857\,i - .8571\,j + .4286\,k)$

$* F = -57.14\,i - 171.42\,j + 85.72\,k$

$r_b = 5i + 6j + 1k$

$-r_a = 0 \quad 0 \quad 0$

$* r_{ab} = 5i + 6j + 1k \text{ (FT)}$

$r_c = 3i + 0j + 4k$

$-r_b = -[5i + 6j + 1k]$

$r_{bc} = -2i - 6j + 3k$

$|r_{bc}| = \sqrt{2^2 + 6^2 + 3^2} = 7$

$e_{bc} = \frac{r_{bc}}{|r_{bc}|} = \frac{-2i - 6j + 3k}{7}$

$e_{bc} = -.2857\,i - .8571\,j + .4286\,k$

$r_c = 3i + 0j + 4k$

$-r_a = -[0 \quad 0 \quad 0]$

$* r_{ac} = 3i + 0j + 4k$

a) $r_{ab} \times F =$

i	j	k	i	j
5	6	1	5	6
-57.14	-171.42	+85.72	-57.14	-171.42

$r_{ab} \times F = 686\,i - 486\,j - 514\,k \text{ (FT·Lbs)}$

b) $r_{ac} \times F =$

i	j	k	i	j
3	0	4	3	0
-57.14	-171.42	+85.72	-57.14	-171.42

$r_{ac} \times F = 686\,i - 486\,j - 514\,k \text{ (FT·Lbs)}$

NOTE HERE $M = T = r_{AB} \times F$